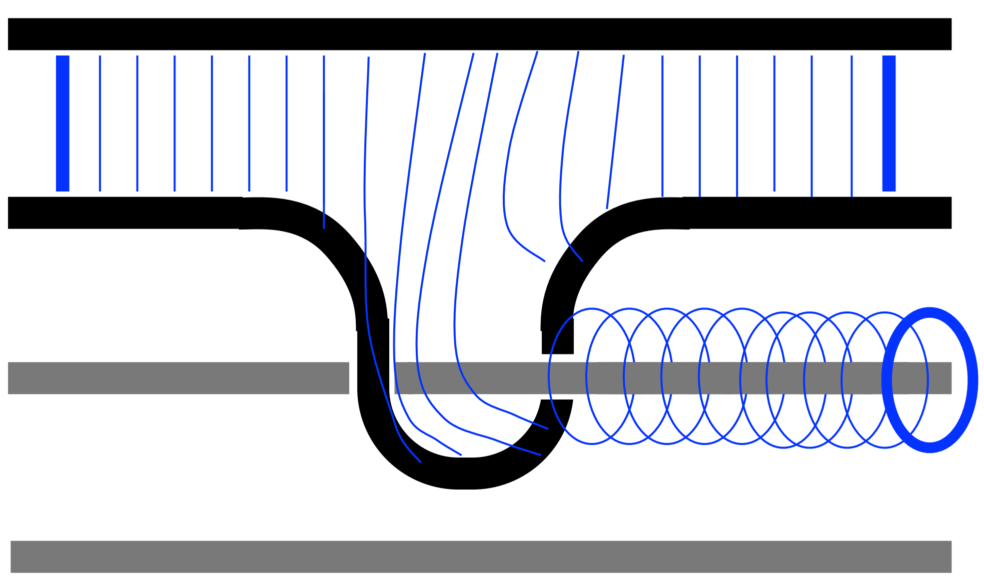
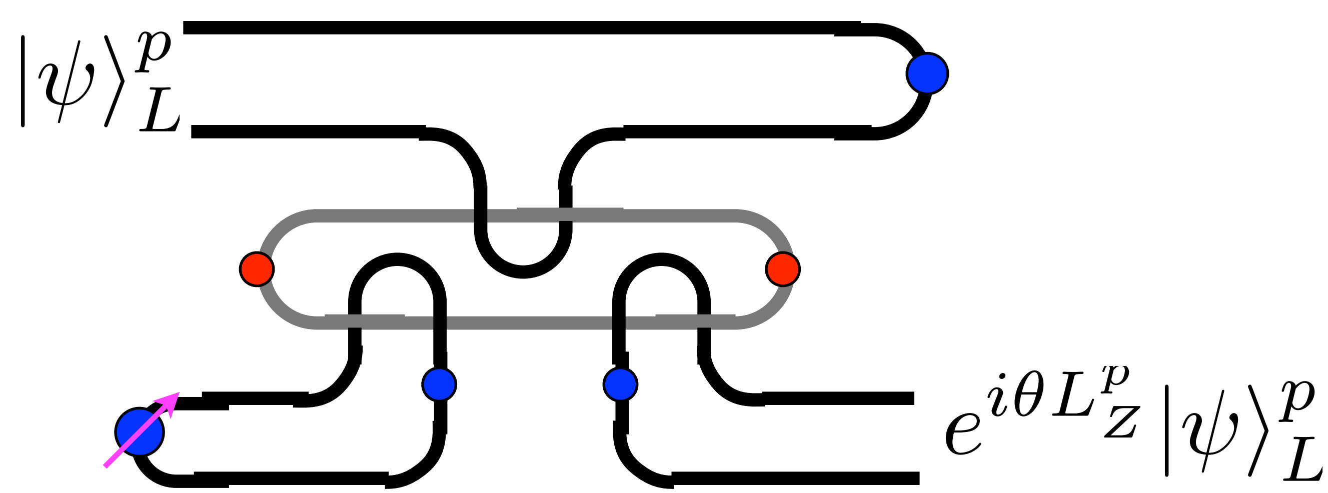
スタビライザー形式と
トポロジカル量子計算

藤井　啓祐





はじめに

1990 X.-G. Wen
Ground-state degeneracy of the fractional quantum Hall states in the presence of a random potential and on high-genus Riemann surfaces:Topological order
Phys. Rev. B 41 9377

1997 A. Kitaev
Surface (Torus) code: Topological quantum memory ← quantum information
Kitaev model: exactly solvable model of topological ordered system
← condensed matter physics
arXiv:970702

2006 R. Raussendorf et al.
Topologically protected fault-tolerant quantum computation
Ann. Phys. 321 2242; NJP 9 199; PRL 98 190504.

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QEC11 Schedule

Monday		Tuesday	
7:00-8:10	Continental breakfast	7:00-8:25	Continental breakfast
7:30-10:00	On-site registration	8:30-9:30	Raymond Laflamme
8:15-8:30	Opening statement (Lidar)	9:30-10:00	Jeongwan Haah
8:30-9:30	Todd Brun	10:00-10:30	Matthew Reed
9:30-10:30	Robert Raussendorf	10:30-10:50	Coffee break
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10:50-11:50	Andrew Landahl	11:30-12:10	Graeme Smith
11:50-12:50	Daniel Lidar	12:10-12:30	Constantin Brif
12:50-2:20	Lunch	12:30-2:00	Lunch
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Legend: Keynote | Tutorial | Best student paper prize talk

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- 最も量子計算機の実現に近いモデル
- 物性物理学と相性が良い

(condensed matter physics)

あらすじ

✓セットアップ

✓スタビライザー形式

✓量子誤り訂正

✓トポロジカル量子メモリ

✓トポロジカル量子計算

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Qubit & Pauli matrices

(1.1) ▶ qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

(1.2) ▶ Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

→ $Y = iXZ$

eg) $X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$ (bit-flip)

$Z|0\rangle = |0\rangle \quad Z|1\rangle = -|1\rangle$ (phase-flip)

$Y|0\rangle = i|1\rangle \quad Y|1\rangle = -i|0\rangle$ (bit&phase-flip + global phase)

note) $|0\rangle, |1\rangle$ are eigenstates of *Z*.

$|\pm\rangle \equiv (|0\rangle \pm |1\rangle)/\sqrt{2}$ are eigenstates of *X*.

Pauli group

(1.3) ▶ *n*-qubit Pauli products:

$$\{\pm 1, \pm i\} \times \{I, X, Y, Z\}^{\otimes n}$$

which forms so-called Pauli group $\mathcal{P}_n$.

eg) 2-qubit Pauli group:

$$\{II, IX, IY, IZ, XI, XX, XY, XZ, YI, YX, YY, YZ, ZI, ZX, ZY, ZZ\} \times \{1, -1, i, -i\}$$

(where $AB \equiv A \otimes B$)

Single-qubit Clifford gates

(1.4) ▶ Clifford operations:= 共役作用（conjugation）のもとで，Pauli products を Pauli products に写すユニタリー演算

$$\begin{array}{ccc} A & \to UAU^\dagger = & B \\ \cap & & \cap \\ \mathcal{P} & & \mathcal{P} \end{array}$$

(1.5) ▶ Hadamard gate *H*: $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$HXH = Z, \ HZH = X$$

eg) $H|0\rangle = |+\rangle, \ H|1\rangle = |-\rangle$

(1.6) ▶ Phase gate *S*: $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

$$SXS^\dagger = Y, \ SYS^\dagger = X, \ SZS = Z$$

Two-qubit Clifford gates

(1.7) ▶ CNOT gate:

$$U_{\mathrm{CNOT}} = |0\rangle\langle 0|_{\mathrm{c}} \otimes I_{\mathrm{t}} + |1\rangle\langle 1|_{\mathrm{c}} \otimes X_{\mathrm{t}}$$

$$\begin{aligned}
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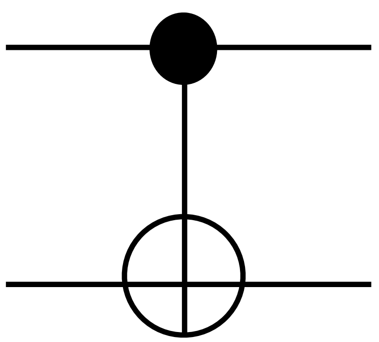

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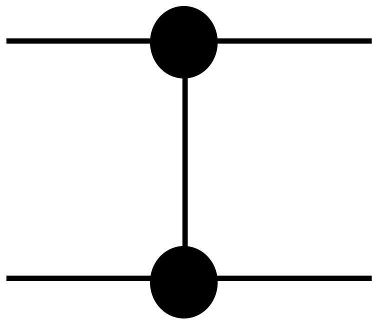

Two-qubit Clifford gates

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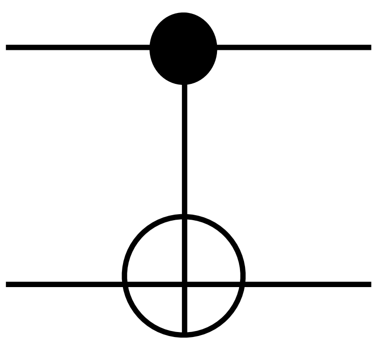

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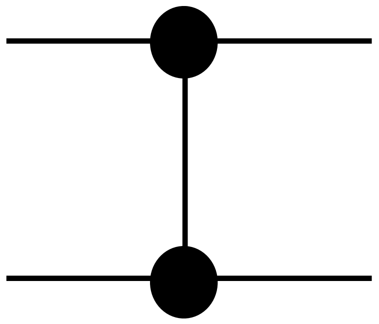

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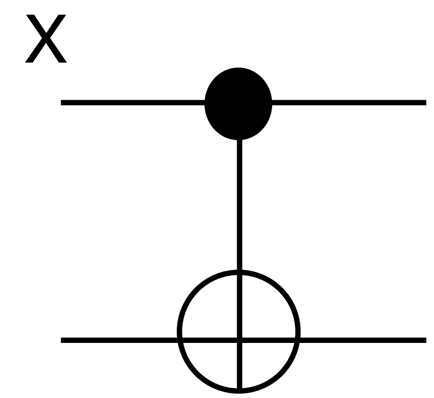

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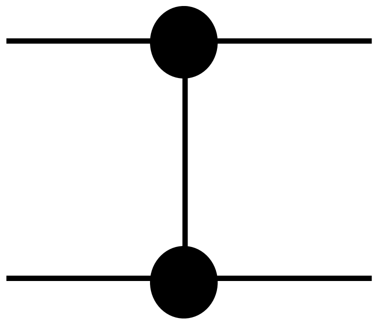

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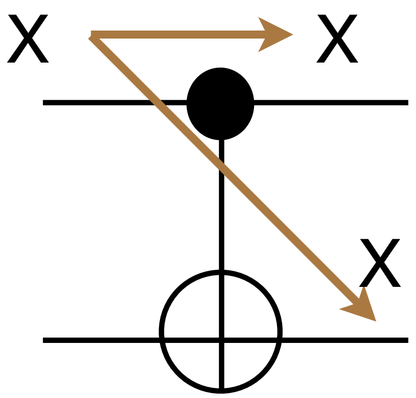

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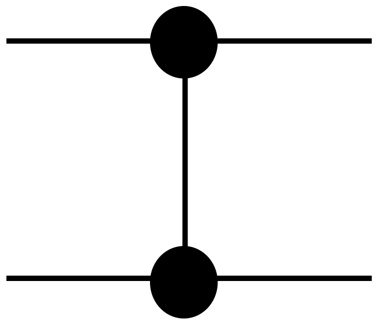

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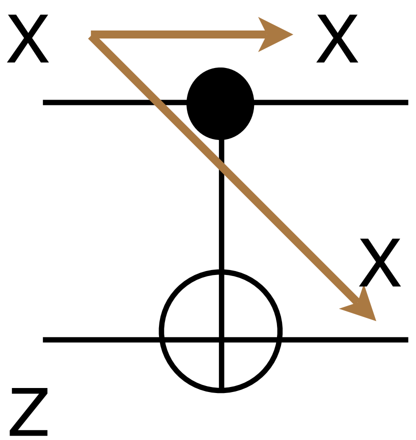

(1.8) ▶ CZ gate:

$$U_{\mathrm{CZ}} = |0\rangle\langle 0|_{\mathrm{c}} \otimes I_{\mathrm{t}} + |1\rangle\langle 1|_{\mathrm{c}} \otimes Z_{\mathrm{t}}$$

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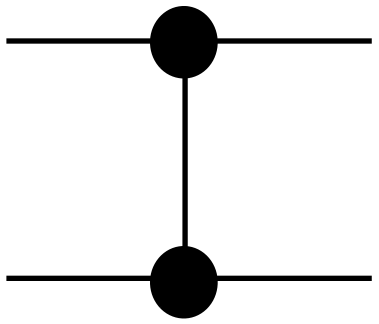

Two-qubit Clifford gates

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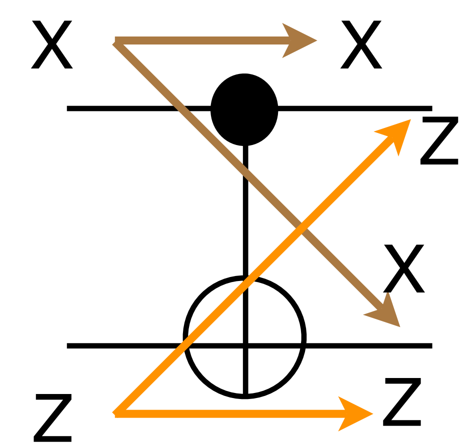

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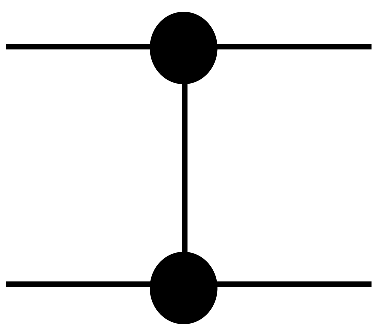

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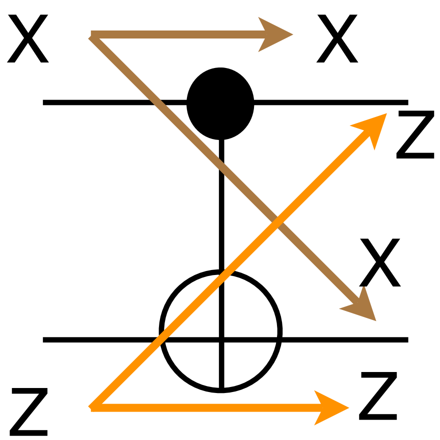

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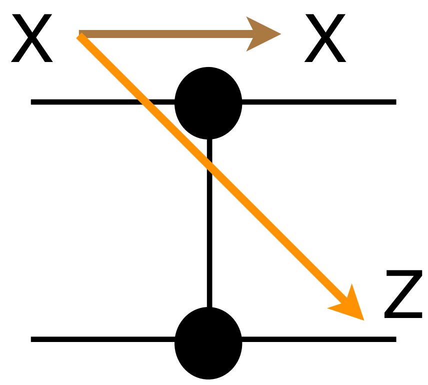

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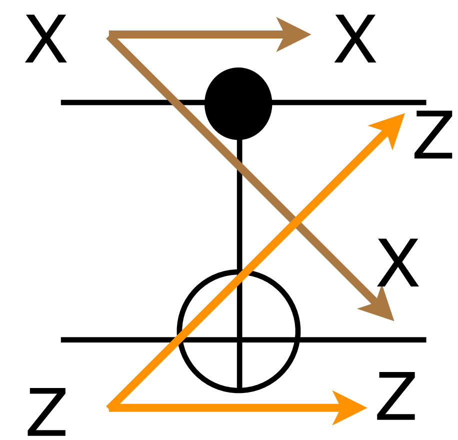

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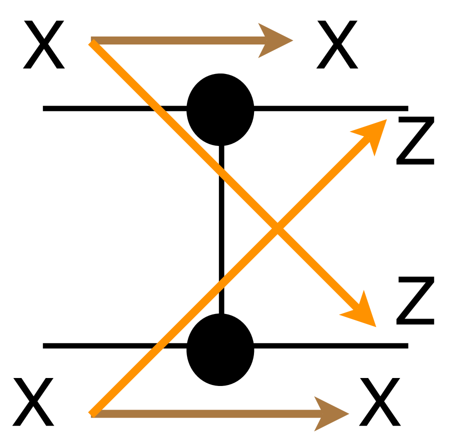

あらすじ

✓セットアップ

qubit, Pauli operator, Clifford gates

✓スタビライザー形式

✓量子誤り訂正

✓トポロジカル量子メモリ

✓トポロジカル量子計算

Stabilizer group

(1.9) ▶ Stabilizer group $\mathcal{S} = \{S_i\}$:= Pauli group の**可換部分群**.

$$[S_i, S_j] = 0$$

eg) $\{IZ, XI, II, XZ\}$ ⟵ no overlap

eg) $\{II, XX, ZZ, -YY\}$ ⟵ even overlap

反可換×2=可換

(1.10) ▶ Stabilizer generators $\mathcal{S}_G = \{\bar{S}_i\}$: stabilizer groupの独立な元の集合.

他の stabilizer generator の積では書けない

eg) $\{IZ, XI\}$ ⟵ no overlap

eg) $\{XX, ZZ\}$ ⟵ even overlap

$\{\bar{S}_i\}$ から生成される Stabilizer groupを $\langle\{\bar{S}_i\}\rangle$ と書くことにする.

eg) $\langle\{XX, ZZ\}\rangle = \{II, XX, ZZ, -YY\}$

Stabilizer state

(1.12) ▶ Stabilizer state $|\Psi\rangle$:

$$S_i|\Psi\rangle = |\Psi\rangle \text{ for all } S_i \in \mathcal{S} \ .$$

stabilizer operator の+1の固有状態

(stabilizer operators はすべて可換)

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n qubit系の次元： 2^n

stabilizer generatorの異なる固有値の固有状態の数： $2^{|\mathcal{S}_G|}$

→ stabilizer generatorの数 $|\mathcal{S}_G|$ が qubit 数 n と等しい場合は状態が一意的に指定される.

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ZZ \ XX	+1	-1
+1	$(\|00\rangle + \|11\rangle)/\sqrt{2}$	$(\|00\rangle - \|11\rangle)/\sqrt{2}$
-1	$(\|01\rangle + \|10\rangle)/\sqrt{2}$	$(\|10\rangle - \|01\rangle)/\sqrt{2}$

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Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$

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eg) $\mathcal{S}_3 = \langle ZZI, IZZ, XXX \rangle$

GHZ state $(|000\rangle + |111\rangle)/\sqrt{2}$

Clifford gates & GK theorem

Clifford gates は Pauli product を Puali product へ写す.

→ stabilizer state を stabilizer state へ写す.

$$S_i|\psi\rangle = |\psi\rangle$$

↓

$$US_iU^\dagger U|\psi\rangle = U|\psi\rangle$$

↓

$$\underline{\bar{S}_i}(U|\psi\rangle) = (U|\psi\rangle)$$

新しいstabilizer group

Gottesman-Knill Theorem

入力状態がPauli基底の状態で，ユニタリー演算が全てClifford演算であり，かつ測定はPauli基底でしか行えない場合，計算結果を古典コンピュータで効率よくシミュレートできる.

n-qubit の stabilizer generator は ($2n \times n$) -bit の古典情報で記述できる.

eg) $XX \rightarrow (1,1|0,0) \quad ZX \rightarrow (0,1|1,0)$

i 番目の X を i 番目 1,　j 番目の Z を (n+j) 番目の 1.

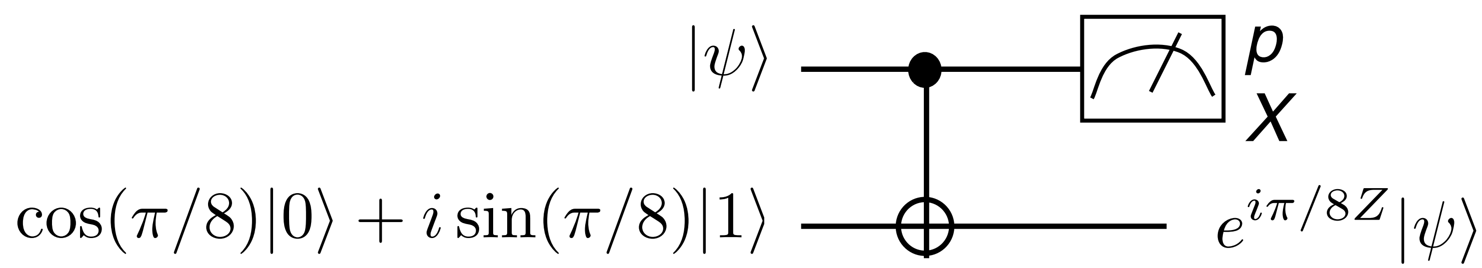
universal quantum computationを実行するためにはさらに何か必要.．．

Magic state distillation

- 一種類のnon-Clifford gateがあればOK

by Solovay-Kitaev theorem

- 特殊なアンシラ状態（magic state）さえあれば，non-Clifford gateが量子テレポーテーションを用いて実行できる.



- ある程度きれいな magic state があれば，Clifford gateを使って，理想的な magic stateをdistillationできる

by Bravyi-Kitaev PRA 71 022316 (2005)

noisy ancilla + Clifford gate = universal

Stabilizer subspace

(1.15) ▶ Stabilizer subspace:

stabilizer generator の数 $|\mathcal{S}_G|$ が qubit 数n よりも小さい場合

stabilizer state は $2^{n-|\mathcal{S}_G|}$ 次元の縮退した部分空間を張る.

eg) $\langle ZZ \rangle$

stabilizer subspace: $\{|00\rangle, |11\rangle\}$

eg) $\mathcal{S}_{\rm bit} \equiv \langle ZZI, IZZ \rangle$

stabilizer subspace: $\{|000\rangle, |111\rangle\}$

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eg) $\mathcal{S}_{\rm bit} \equiv \langle ZZI, IZZ \rangle$

stabilizer subspace: $\{|000\rangle, |111\rangle\}$

(1.16) ▶ Logical operators:

$L_i^X L_i^Z = -L_i^Z L_i^X$ と $[L_i^A, S_j] = 0$ を満たす$n - |\mathcal{S}_G|$ 組の演算子, L_i^X, L_i^Z

反可換 **全てのstabilizerと可換** $(i = 1, \cdots, n - |\mathcal{S}_G|)$

eg) $\langle ZZ \rangle$

$L_1^X = XX, \;\; L_1^Z = IZ$

L_1^{XX} の固有状態 $|00\rangle \pm |11\rangle$

L_1^Z の固有状態 $|00\rangle, |11\rangle$

eg) $\mathcal{S}_{\rm bit} = \langle ZZI, IZZ \rangle$

$L_1^X = XXX, \; L_1^Z = ZII$

Stabilizer code

(1.17) ▶ Logical basis:

$\langle S_j, (-1)^{i_1} L_1^Z, \cdots, (-1)^{i_k} L_k^Z \rangle$ によって stabilize される状態を

logical computational basis $|i_1, \cdots, i_k\rangle_L$ として定義する.

($k = n - |\mathcal{S}_G|$)

注）L_i^Z, L_i^X は logical basis 上のlogical Pauli operatorになっている.

$$L_k^Z |i_1, \cdots, i_k\rangle_L = (-1)^{i_k} |i_1, \cdots, i_k\rangle_L$$

$$L_j^X |i_1, \cdots, i_k\rangle_L = |i_1, \cdots, i_j \oplus 1, \cdots, i_k\rangle_L$$

$$\left(L_j^Z (L_j^X |i_1, \cdots, i_k\rangle_L) = -L_j^X (-1)^{i_j} |i_1, \cdots, i_k\rangle_L = (-1)^{i_j \oplus 1} (L_j^X |i_1, \cdots, i_k\rangle_L) \right)$$

eg) $\langle ZZI, IZZ \rangle,\ L_1^Z = ZII \rightarrow |0\rangle_L = |000\rangle,\ |1\rangle_L = |111\rangle$

$$L_1^Z |1\rangle_L = -|1\rangle_L$$

$$L_1^X |0\rangle_L = |1\rangle_L$$

(1.18) ▶ Stabilizer code:

stabilizer groupと logical operator によって定義される量子誤り訂正符号.

あらすじ

- ✓ セットアップ

  qubit, Pauli operator, Clifford gates

- ✓ スタビライザー形式

  stabilizer group, state, subspace, logical operator, magic state

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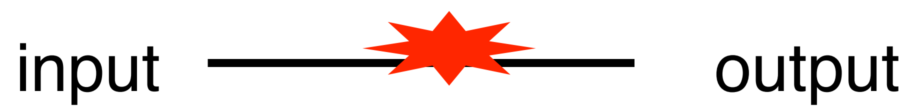
Quantum error correction

(1.19) ▶Pauli error:



bit-flip error: $(1-p)\rho + pX\rho X$

phase-flip error: $(1-p)\rho + pZ\rho Z$

depolarizing error: $(1-p)\rho + p/3(X\rho X + Y\rho Y + Z\rho Z)$

▶Pauli errorの符号空間への作用:

$\langle S_i \rangle$ によって定義されるstabilizer codeにあるPauli operator (Pauli product) がエラーとして作用した時，以下の２つの場合が考えられる

- $[A, S_i] = 0$ 全てのstabilizerと可換.
  - → A がstabilizer である．logical stateを変化させない．
  - → A がlogical operator である．logical stateを変化させる．
- $[A, S_j] \neq 0$ stabilizerのどれかと反可換 → 状態がstabilizer subspaceの外（直交補空間）に出る．

Quantum error correction

ex)

$\alpha|000\rangle + \beta|111\rangle$ —IZZ→ $\alpha|000\rangle + \beta|111\rangle$ 何もなかった。

$\alpha|000\rangle + \beta|111\rangle$ —IIZ→ $\alpha|000\rangle - \beta|111\rangle$ 状態が壊れた。

$\alpha|000\rangle + \beta|111\rangle$ —IIX→ $\alpha|001\rangle + \beta|110\rangle$ 状態が直交補空間へ飛んだ。

(1.20) ▶Syndrome subspace and error syndrome:

$\langle s_i S_i \rangle$ （ $s_i = \pm 1$ ）によって定義される $\langle S_i \rangle$ の直交補空間を

$\mathcal{H}(s_1, \cdots, s_{n-k})$ とし, $(s_1, \cdots, s_{n-k})$ を error syndromeと呼ぶ.

(1.21) eg) $\mathcal{S}_{\mathrm{bit}} = \langle ZZI, IZZ \rangle$

IZZ \ ZZI	+1	-1
+1	$\mathcal{H}(+1,+1)$ $\|000\rangle, \|111\rangle$	$\mathcal{H}(+1,-1)$ $\|100\rangle, \|011\rangle$
-1	$\mathcal{H}(-1,+1)$ $\|001\rangle, \|110\rangle$	$\mathcal{H}(-1,-1)$ $\|010\rangle, \|101\rangle$

Quantum error correction

(1.22) ▶Quantum error correction:

(1) どのsyndrome subspaceに状態がいるかを知る(= error syndrome を知る)
→ Stabilizer generator を観測量として射影測定を行う=syndrome measurement.

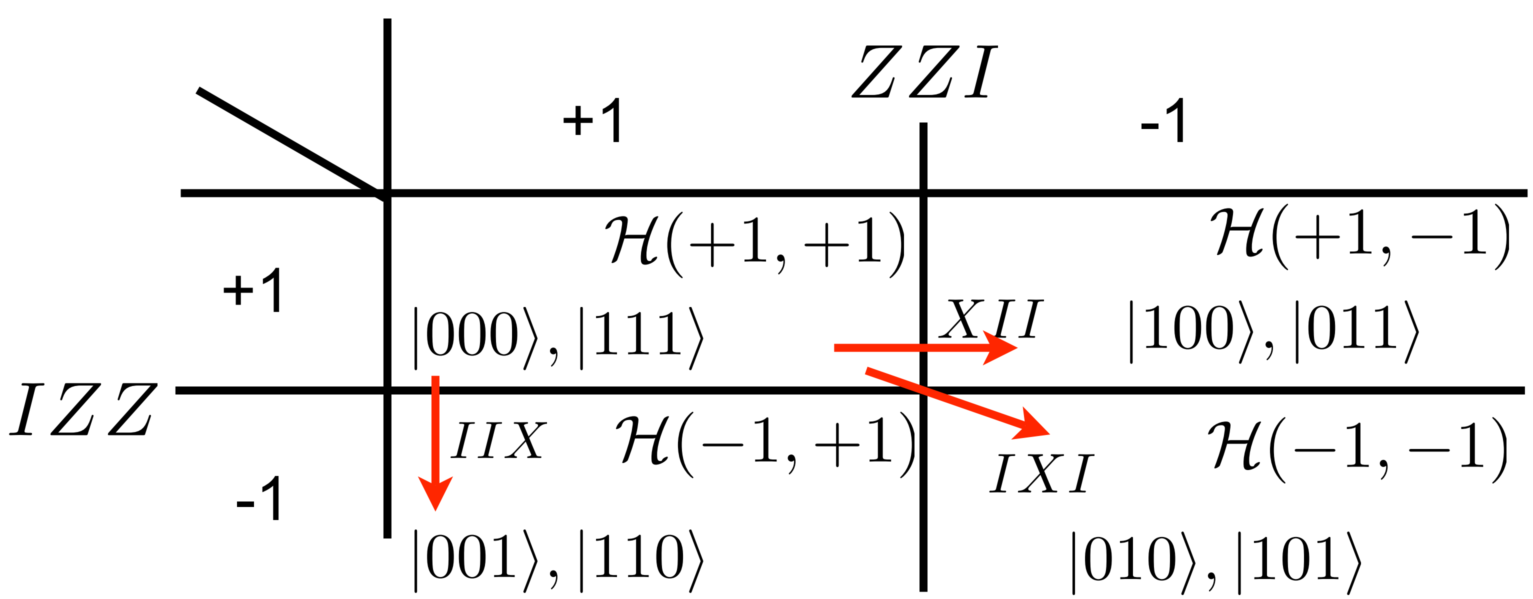
(2) error syndrome からどのようなエラーが起きているか推定し，訂正する.

eg) bit-flip error が3つのqubitに独立に作用するような場合を考える.

もし，error syndrome (+1,-1)が得られたとき，

XII with probability ~p or IXX with probability ~p^2

most likely

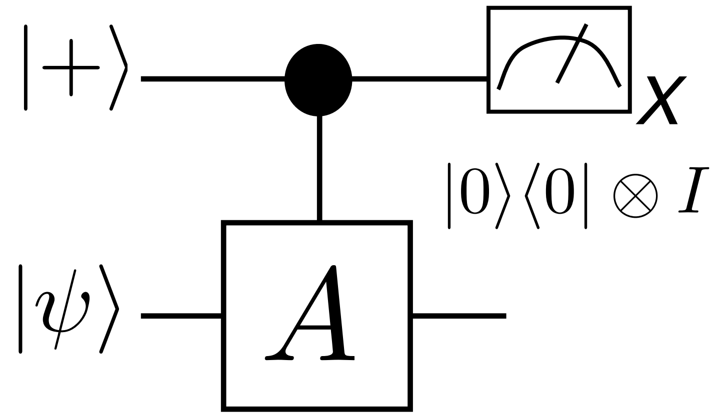


エラーの発生確率 p が十分小さければこの操作によってeffectiveなエラー確率が減少する.

Syndrome measurement

(1.23) ▶Indirect projective measurement:

*A*を固有値 ±1 のエルミート演算子とする.



$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes A$$

projective measurement of *A*

$$(|0\rangle_m|\psi\rangle + |1\rangle_m A|\psi\rangle)/\sqrt{2}$$

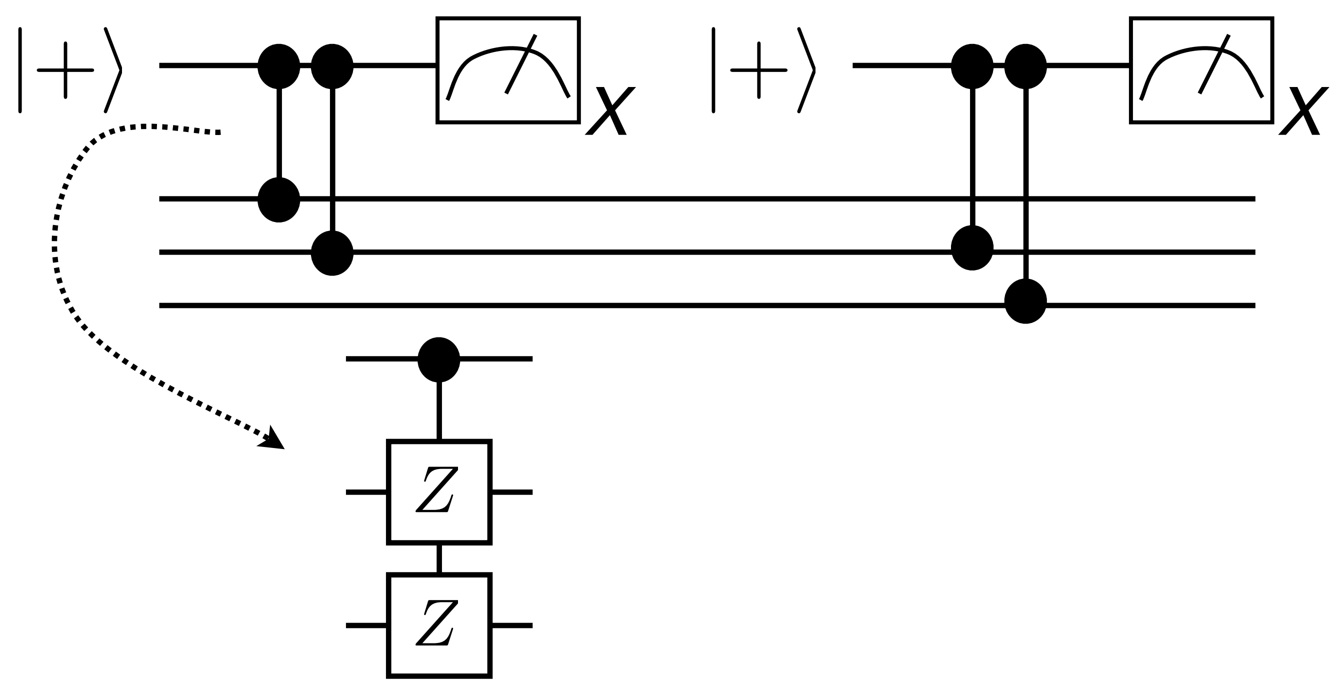
$$\downarrow |+\rangle\langle +|$$

$$|+\rangle \underline{\left(\frac{I+A}{2}\right)} |\psi\rangle$$

*A*の+1の固有状態への

射影演算子

eg) $\mathcal{S}_{\rm bit} = \langle ZZI, IZZ \rangle$



Steane 7-qubit code

1つのqubitに作用する*X* error もしくは *Z* errorを訂正できる.

Stabilizer generators:

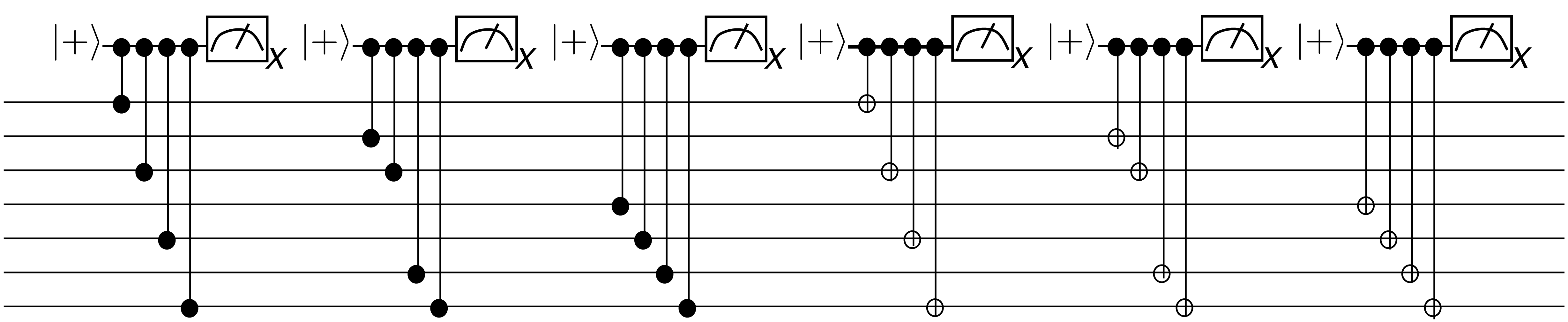
$$S_1 = ZIZIZIZ \qquad S_4 = XIXIXIX$$
$$S_2 = IZZIIZZ \qquad S_5 = IXXIIXX$$
$$S_3 = IIIZZZZ \qquad S_6 = IIIXXXX$$

Syndrome subspaceの数は 2^6=64
errorの種類は (1+7)(1+7)=64　→異なるエラーが異なる直交補空間に対応している.

一般に *t* 個のエラーを訂正するためには

$$\left[\sum_{i=0}^{i=k} \begin{pmatrix} n \\ t \end{pmatrix}\right]^2 \leq 2^{n-1}$$

等号成立は (n,t)=(7,1),(23,3)....



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stabilizer group, state, subspace, logical operator, code

✓量子誤り訂正

syndrome measurement, indirect measurement

✓トポロジカル量子メモリ

✓トポロジカル量子計算

Topological quantum computation

Abelian

Topological quantum computation with Abelian anyon on the surface which consists of qubits.

surface (torus) code(memory):

Kitaev, Annals Phys. **303**, 2 (2003)

topological quantum computation:

Raussendorf-Harrington-Goyal, Annals Phys. **321**, 2242 (2006)

Raussendorf-Harrington-Goyal, NJP **9**, 199 (2007)

Raussendorf-Harrington, PRL **98**, 190504 (2007)

what we want to study

non-Abelian

Topological quantum computation with non-Abelian anyon.

quantum memory:
ν=5/2 fractional quantum Hall state
(thought to be non-Abelian anyon)

universal quantum computation:
Fibonacci anyon

Lecture Notes for Physics 219: Quantum Computation by Preskill

Sarma-Freedman-Nayak Physics Today 32 July 2006

Sarma-Freedman-Nayak Rev. Mod. Phys. **80,** 1083 (2008).

Surface (torus) code

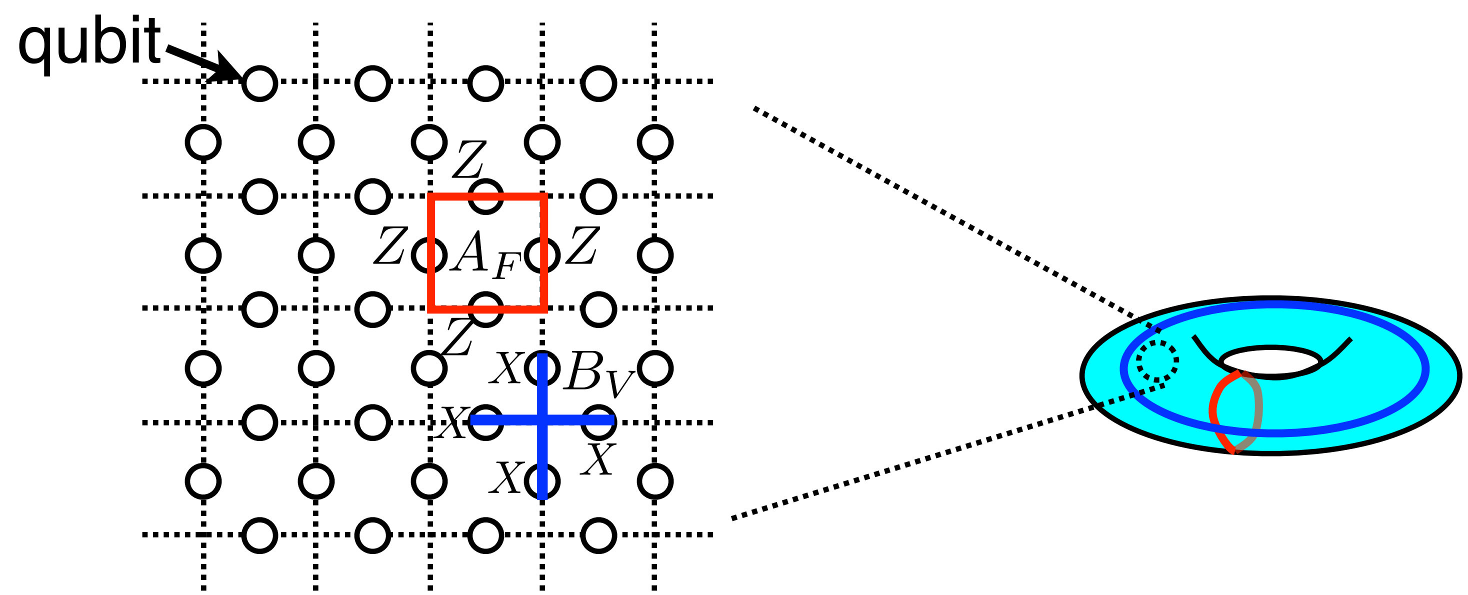
by A. Kitaev '97 (arXiv:9707021)

(2.1) ▶ Stabilizer of the surface code:

The face and vertex operators are defined for all faces and vertexes.

$$A_F = \bigotimes_{i \in F} Z_i \quad B_V = \bigotimes_{j \in V} X_j$$

note) A_F and B_V are commutable, since each face and vertex share 0 or 2 qubits. Thus $\{A_F, B_V\}$ is a stabilizer group.



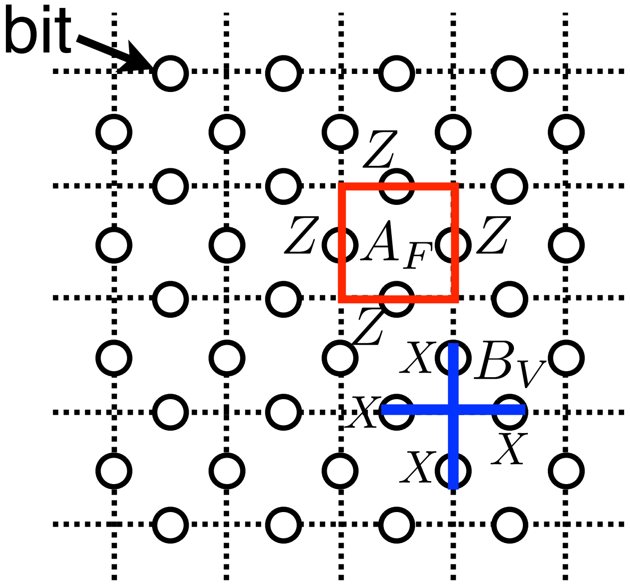
(2.2) ▶ Surface code state $|\Psi\rangle$:

$A_F|\Psi\rangle = |\Psi\rangle$ and $B_V|\Psi\rangle = |\Psi\rangle$ for all *F* and *V*.

Surface (torus) code

by A. Kitaev '97 (arXiv:9707021)

(2.3) ▶the number of the logical qubits encoded on the torus:



Recall (1.15).

# of qubits (edges) in $N \times N$ torus: $2N^2$

# of stab. generators (faces & vertexes): $2N^2 - 2$

note) -2 comes from the fact that $\prod_F A_F = I^{\otimes 2N^2}$ and $\prod_V B_V = I^{\otimes 2N^2}$, and one face and one vertex operator are not independent.

# of logical qubits: 2

(2.4) ▶In general.....

(face)+(vertex)-(edge)=2-2g where g is the genus of the surface.

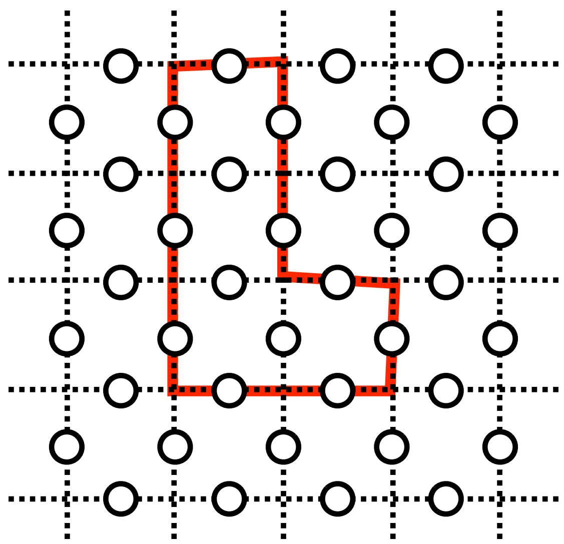
Euler characteristic

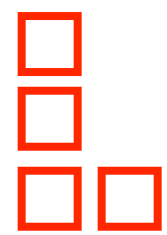
# of logical qubits → (edge)-[(face)+(vertex)-2]=2g

Trivial loop operator

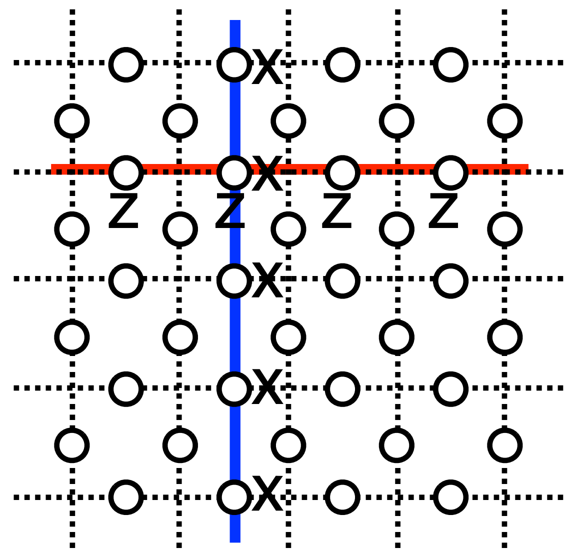
(2.5) ▶ Trivial loop operators act on the code space trivially:

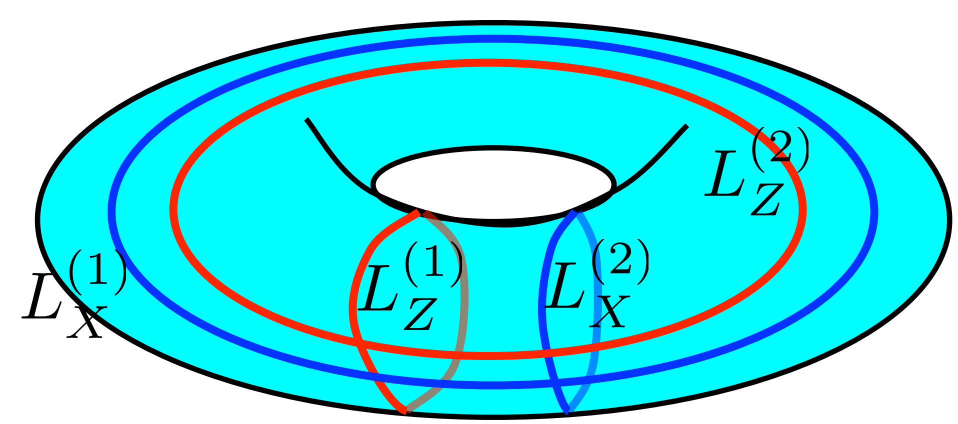
= there is no hole or defect inside the loop



Since the loop operator  is a product of , it is also a stabilizer operator. Thus its multiplication does not change the code state.

(2.6) ▶ Non-trivial loop operators commute with all stabilizer operators, but they are not products of stabilizer operators. Thus non-trivial loop operators are logical operators (recall (1.16)), which represent logical qubits.

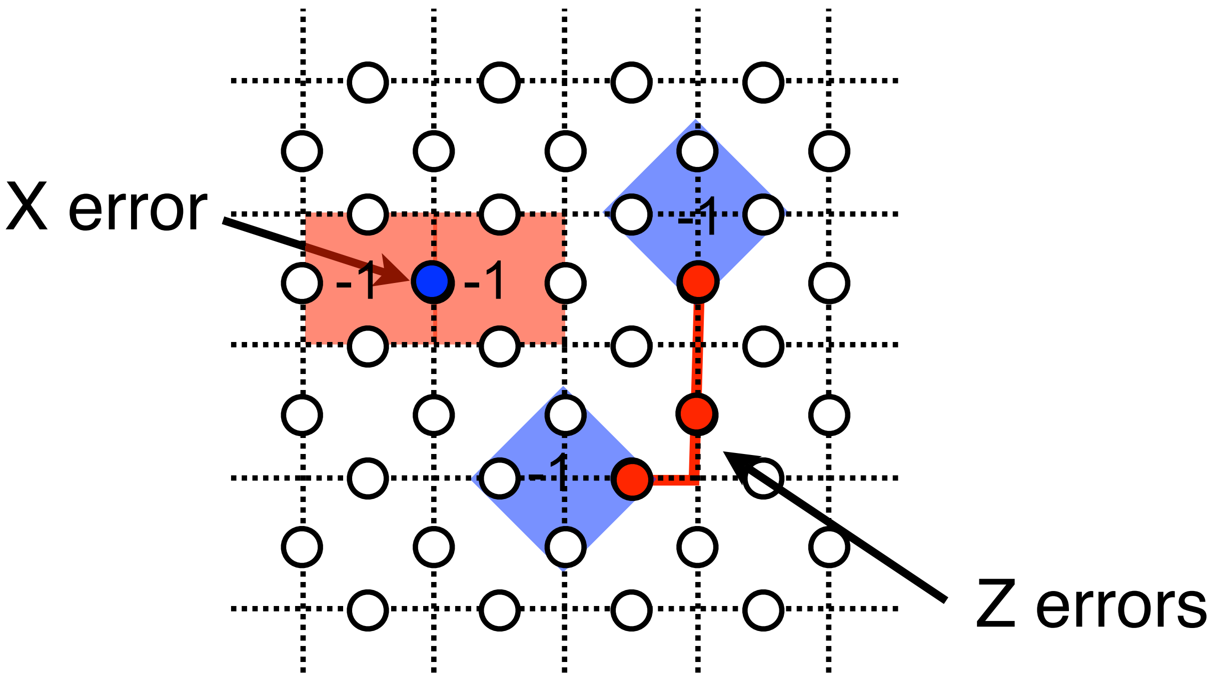




How to correct error

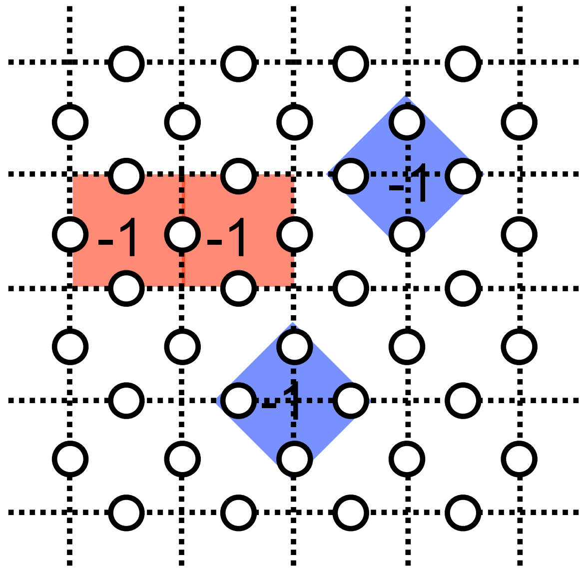
(2.7) ▶Error syndromes:

Incorrect error syndromes are found at boundaries of an error chain, since Pauli operators on the boundaries anti-commute with stabilizers (recall (1.19)).



(2.8) ▶Error correction:

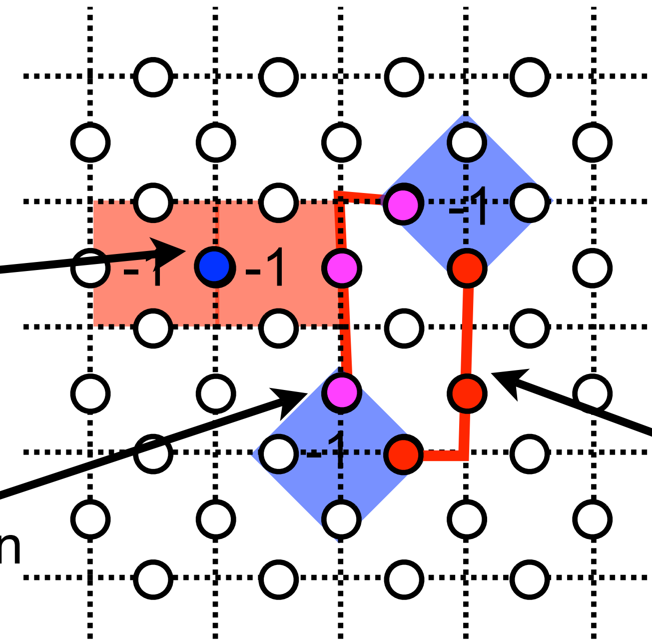
Infer the most-likely locations of errors conditioned on the error syndrome (recall (1.22))
→ minimum-weight-perfect matching algorithm

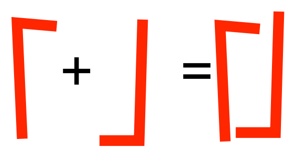


How to correct error

(2.9)

▶ Successful error correction:



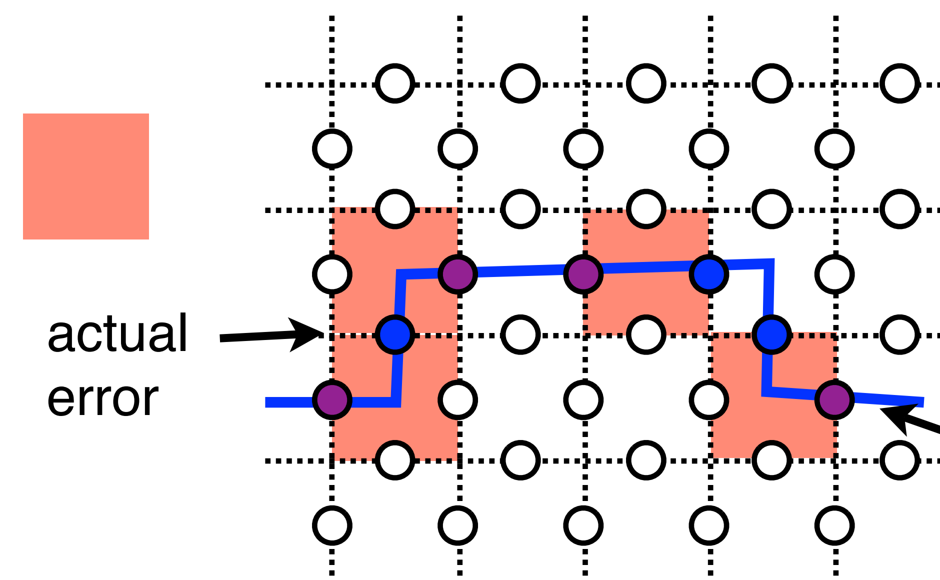
note) Since  trivial loop operator, the recovery operation successfully corrects the error.

(2.10)

▶ logical error:

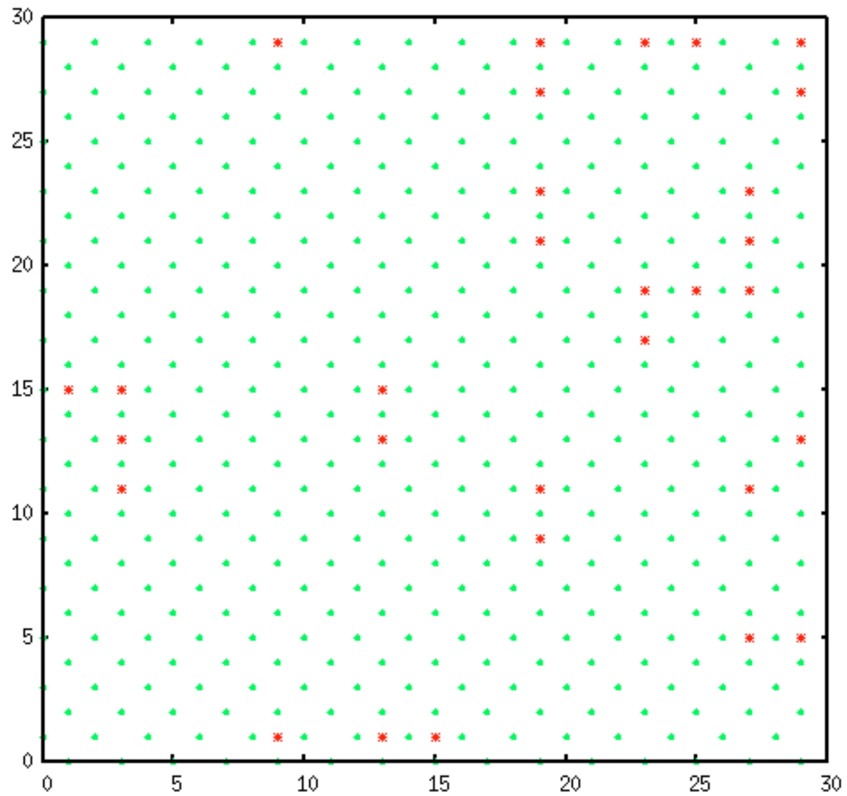
If errors are too dense, recovery operation results in a nontrivial loop operator, which changes the code state.

The critical error probability, below which topological error correction succeeds, is ~0.11, which is very close to the quantum Gilbert-Varshamov bound in the limit of zero asymptotic rate.

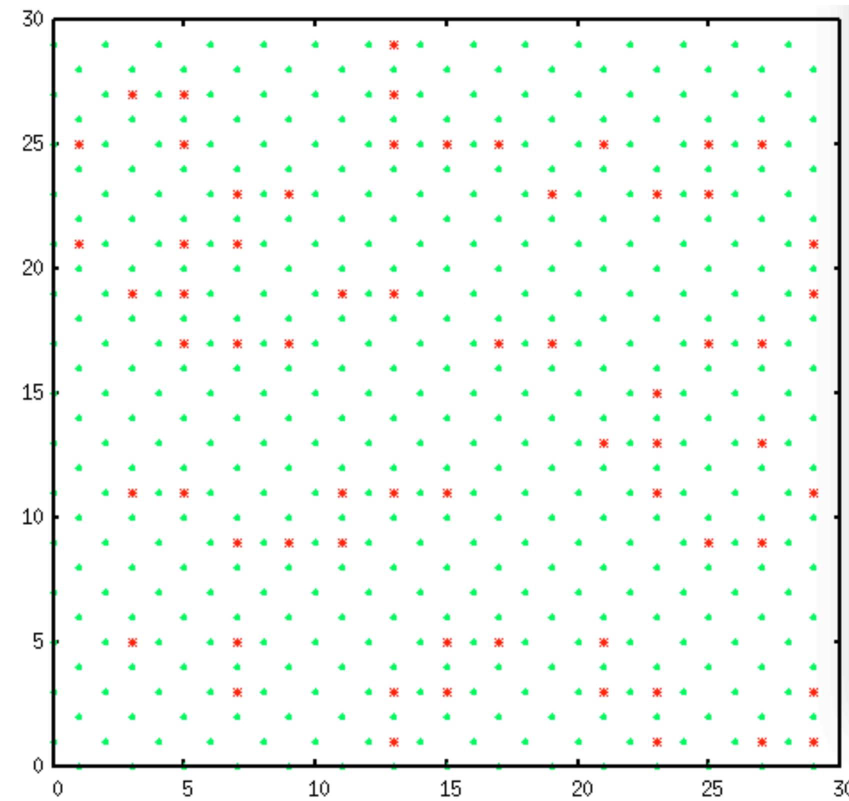


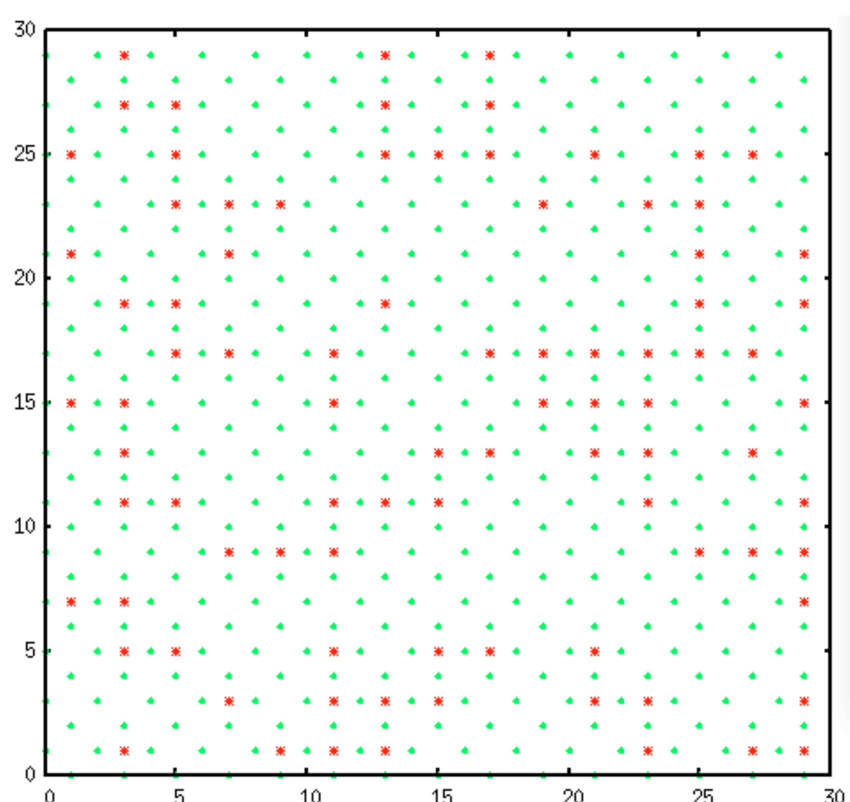
Threshold value

p=3%



p=10%





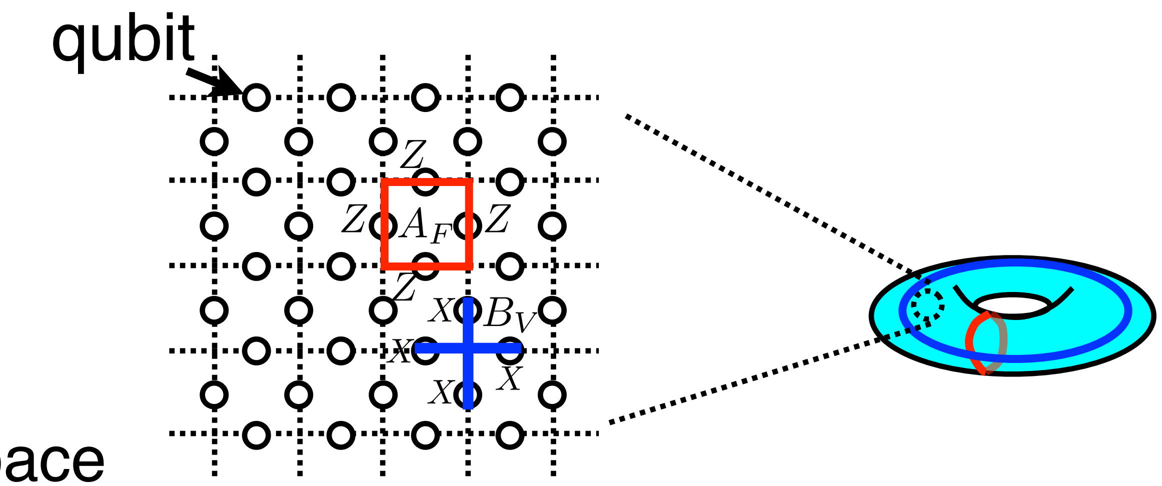
p=15%

Random-bond Ising modelにおけるNishimori line上の相転移温度とSurface codeの誤り訂正限界とに厳密な対応がある.

Kitaev model

Hamiltonian:

$$H = -J\left(\sum_F A_F + \sum_V B_V\right)$$



- translationally invariant
- ground state subspace = stabilizer subspace
- topological order = ground state degeneracy cannot be distinguished by local operator (cannot be explained by Landau's sponteneous symmetry breaking)
- anyonic excitation

絵を入れる

- String-net condensate by X.-G. Wen

$$|\Psi_{\rm vac}\rangle = \bigotimes_V \left(\frac{I + B_V}{2}\right)|00\cdots 0\rangle$$

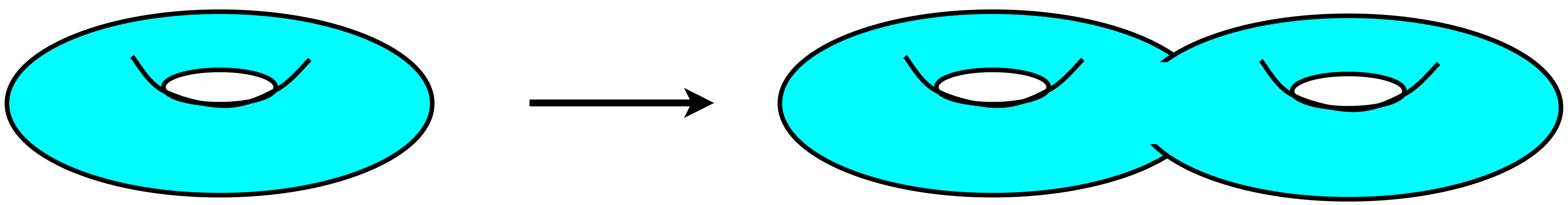
あらすじ

- ✓ セットアップ
  qubit, Pauli operator, Clifford gates
- ✓ スタビライザー形式
  stabilizer group, state, subspace, logical operator, code
- ✓ 量子誤り訂正
  syndrome measurement, indirect measurement
- ✓ トポロジカル量子メモリ
  surface code, trivial & non-trivial loop operator
- ✓ **トポロジカル量子計算**

How to increase logical qubits

(2.11) ▶ Changing the topology of the surface:

recall (2.4): # of logical qubits is $2g$



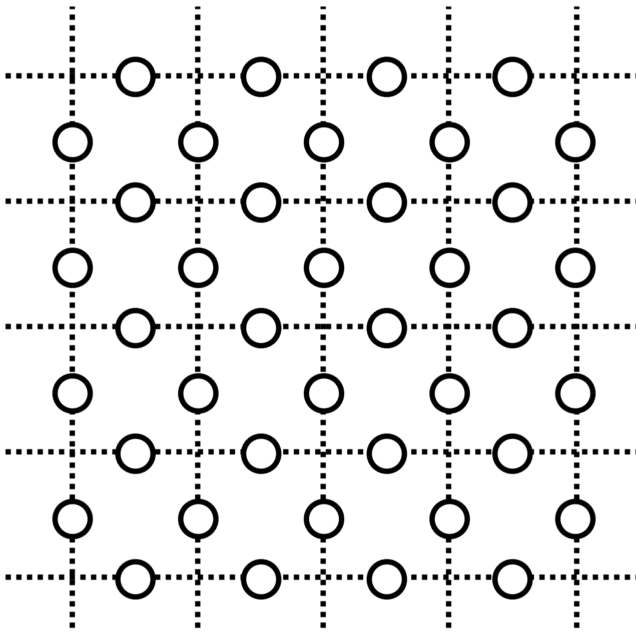
But, this approach is somewhat complicated. Is there any way to increase logical degree of freedom systematically?

How to increase logical qubits

(2.12) ▶Injecting the defects on the surface:

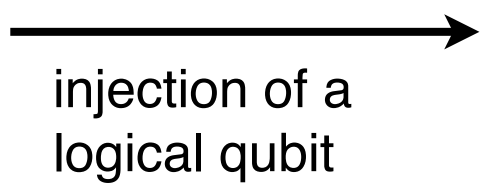
Consider a plane surface instead of torus.

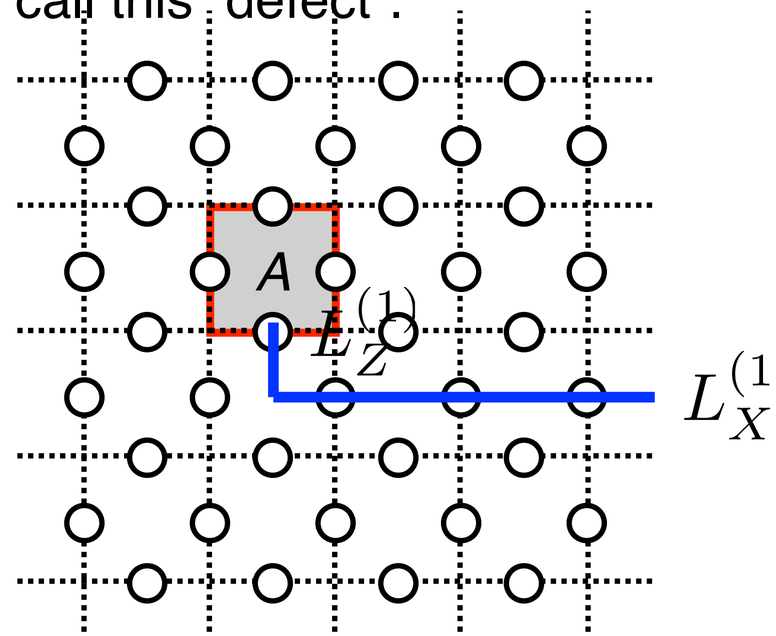
A stabilizer *A* is removed from the stabilizer gp.
We call this “defect”.



# of qubits: 2N(N-1)
# of stabilizers: $(N-1)^2+N^2-1$
→ stabilizers on the plane surface defines a state (not subspace).→”vacuum”

$|\Psi_{\rm vac}\rangle$





The operator *A* is commutable with all stabilizers, thus it is a logical operator, say $L_Z^{(1)}$.

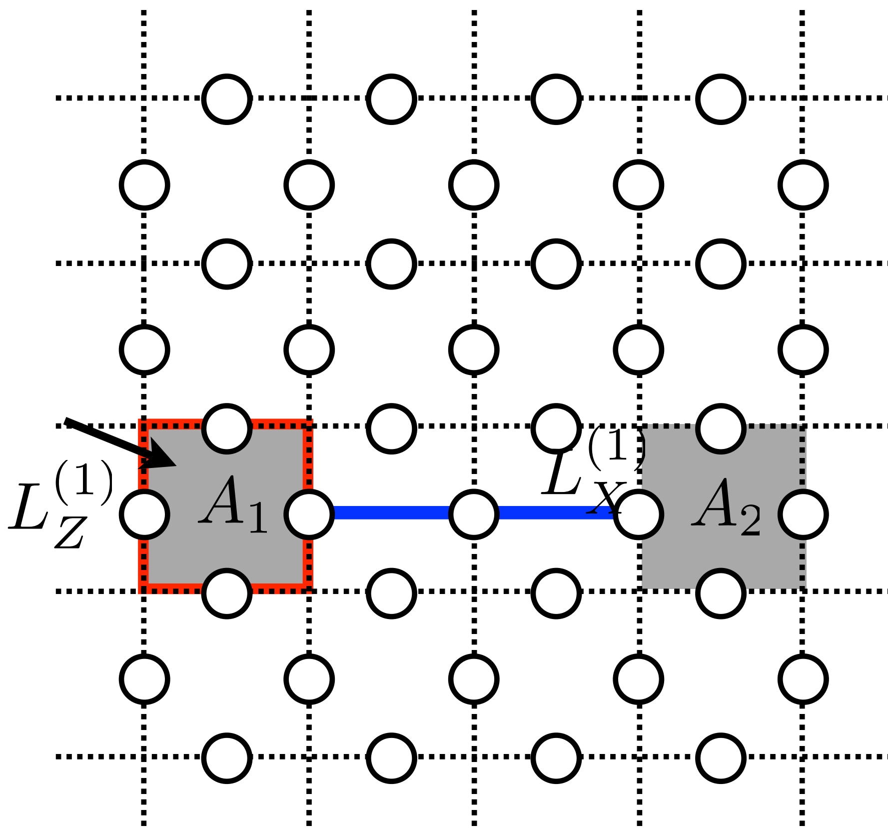
We can also find $L_X^{(1)}$.

Primal defect pair

(2.13) ▶Pairing the defects as logical qubit:

Boundary might be far away from the defects. If we inject a lot of defects, the logical operators might be complicated.....

→ We introduce a logical by creating a defect pair.



A_1 and A_2 are removed from stabilizer group, but A_1A_2 is still an element of stabilizer group.

The $L_Z^{(1)}$ and $L_X^{(1)}$ are both commutable with all stabilizers.

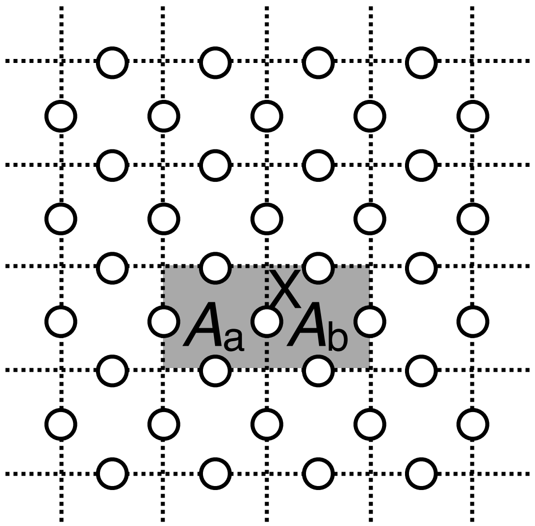
The $L_Z^{(1)}$ and $L_X^{(1)}$ anti-commute, since they share one qubit.

→ The defect pair represents a logical qubit.

Defect pair creation (state preparation)

(2.14) ▶How to create the defect pair:

Measure a qubit in the X basis.



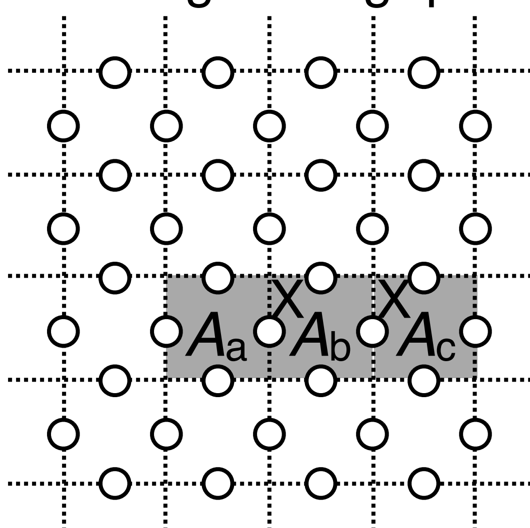
A_a and A_b are removed from the stabilizer group, since they are anti-commute with X.

But, A_aA_b is still a stabilizer.

Defect pair creation (state preparation)

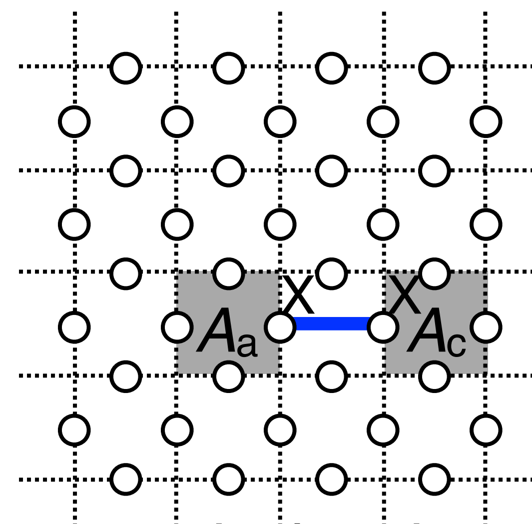
(2.15) ▶How to move the defect:

Measure the neighboring qubit in the X basis.

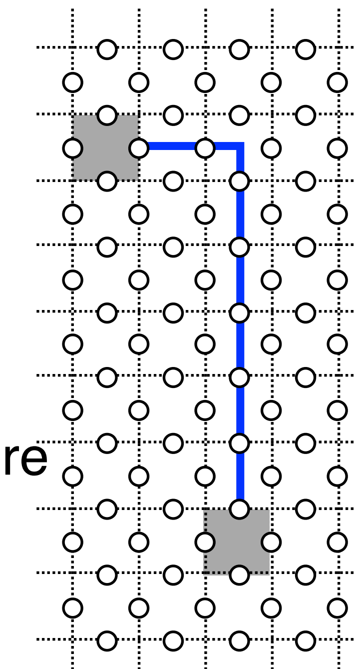


separation

Measure the A_b, and it is revived as a stabilizer.

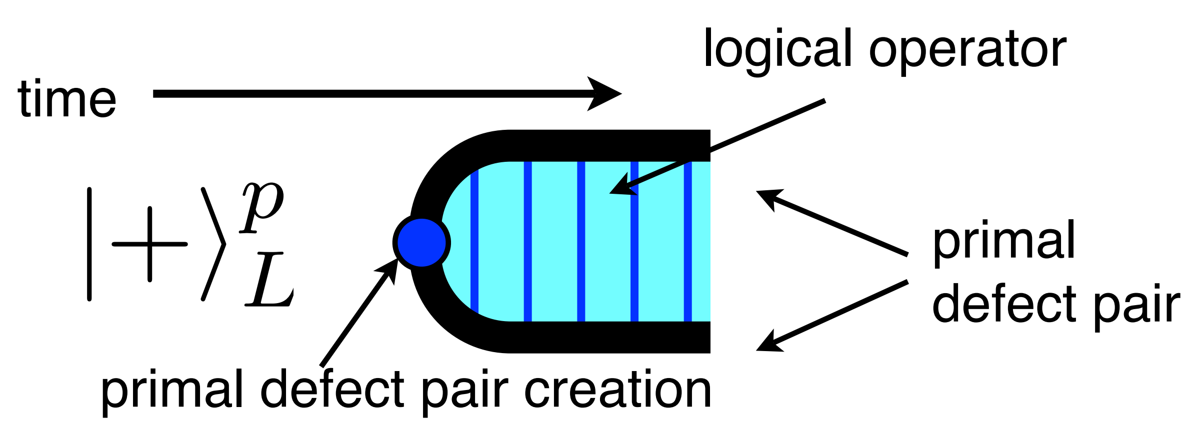


repeating this procedure



A_c is removed from the stabilizer group, but $A_aA_bA_c$ is still a stabilizer.

Since the state is the eigenstate of XX, which is the logical X operator, $|+\rangle_L = (|0\rangle_L + |1\rangle_L)/\sqrt{2}$ is now injected.



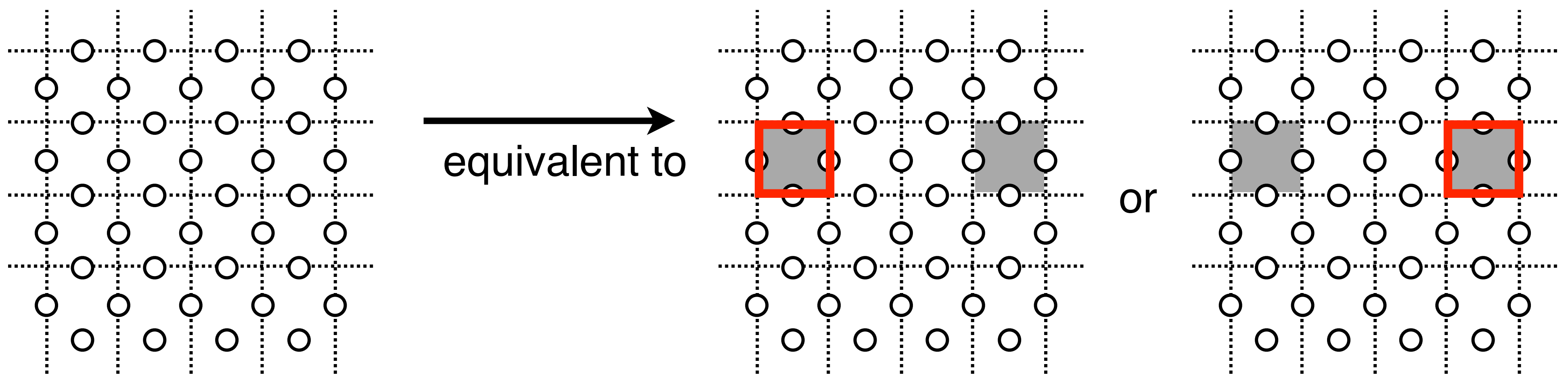
Defect pair creation (state preparation)

(2.16) ▶How to prepare logical Z eigenstate:

The defect pair created in (2.14) & (2.15) is the logical X eigenstate.

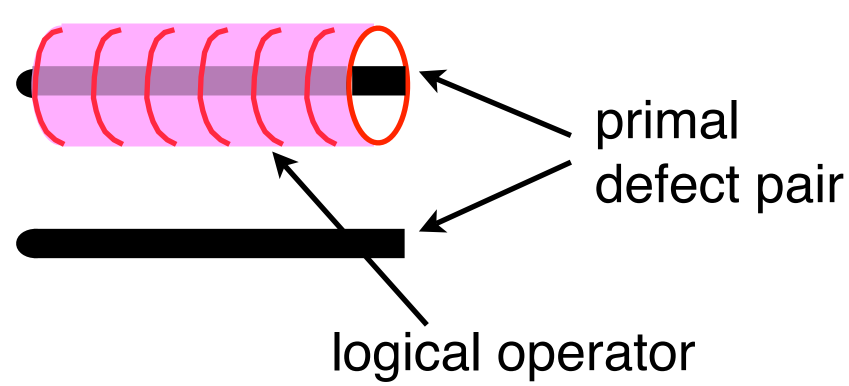
How is the Z eigenstate prepared?

→ The logical Z operator $L_Z^{(1)}$ is the removed face operator. Thus the eigenstate of $L_Z^{(1)}$, that is the stabilizer state before the defect injection.



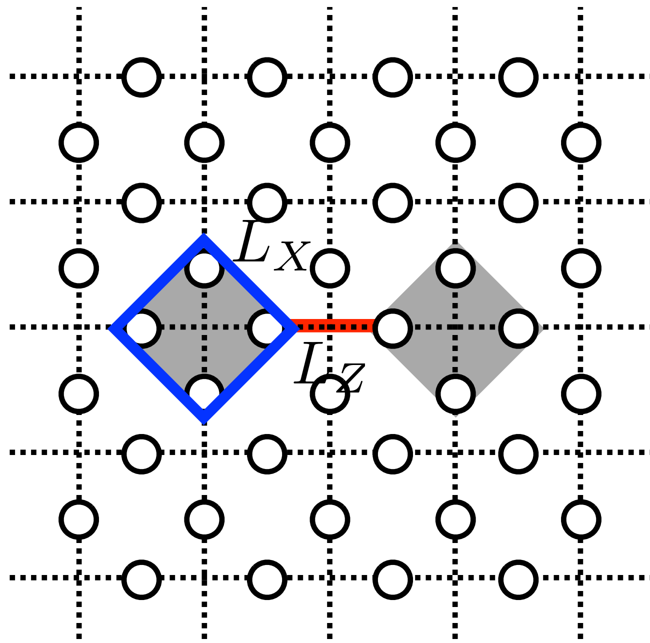
$|\Psi_{\mathrm{vac}}\rangle = |0\rangle_L^p$

$|0\rangle_L^p$



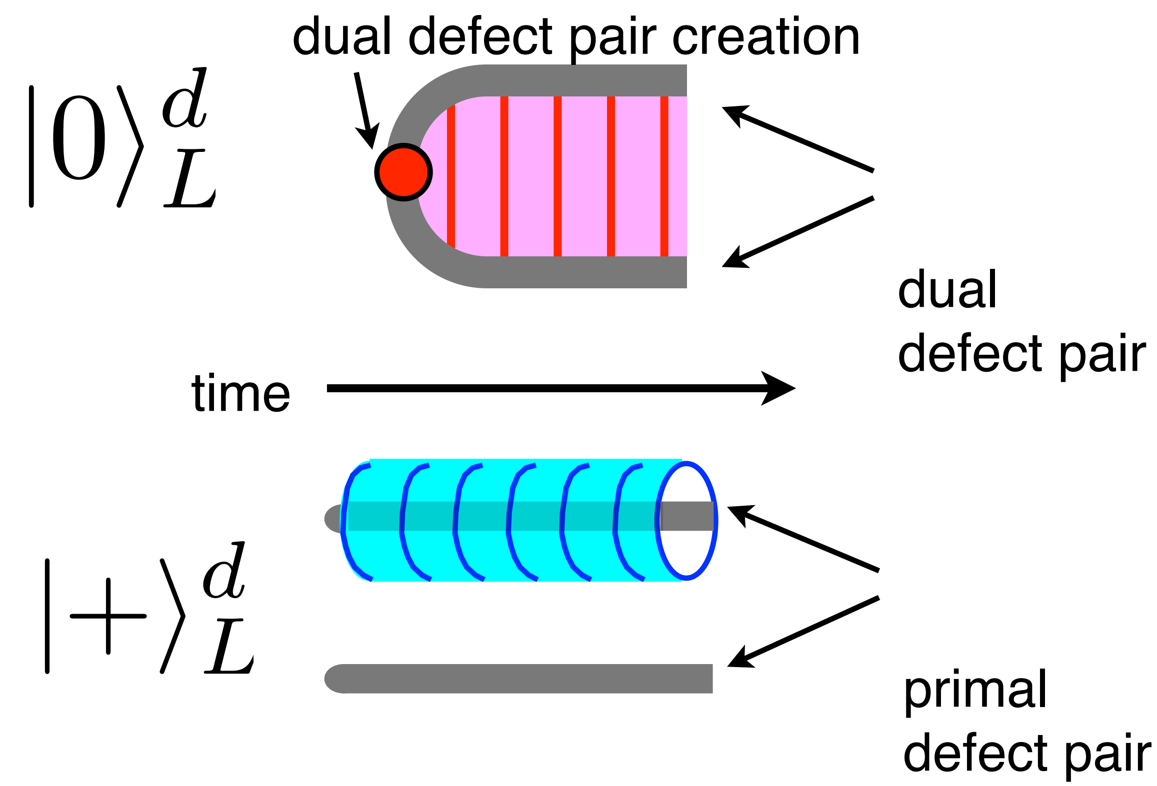
Dual defect pair creation (state preparation)

(2.17) ▶ We can also consider the dual defect pair:



Defect pair creation (with Z basis measurement), separation can be done similarly to the primal defect (2.14,15).

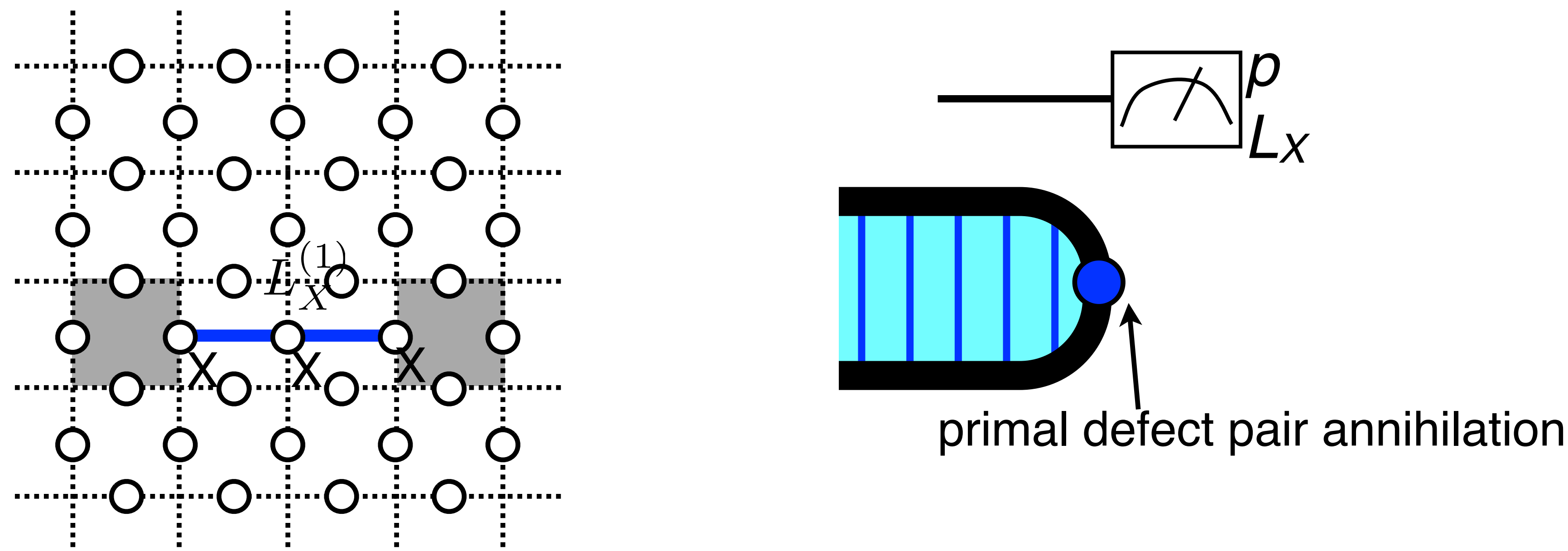
Similarly to (2.16), $|\Psi_{\rm vac}\rangle = |+\rangle_L^d$.



Defect pair annihilation (Logical qubit measurement)

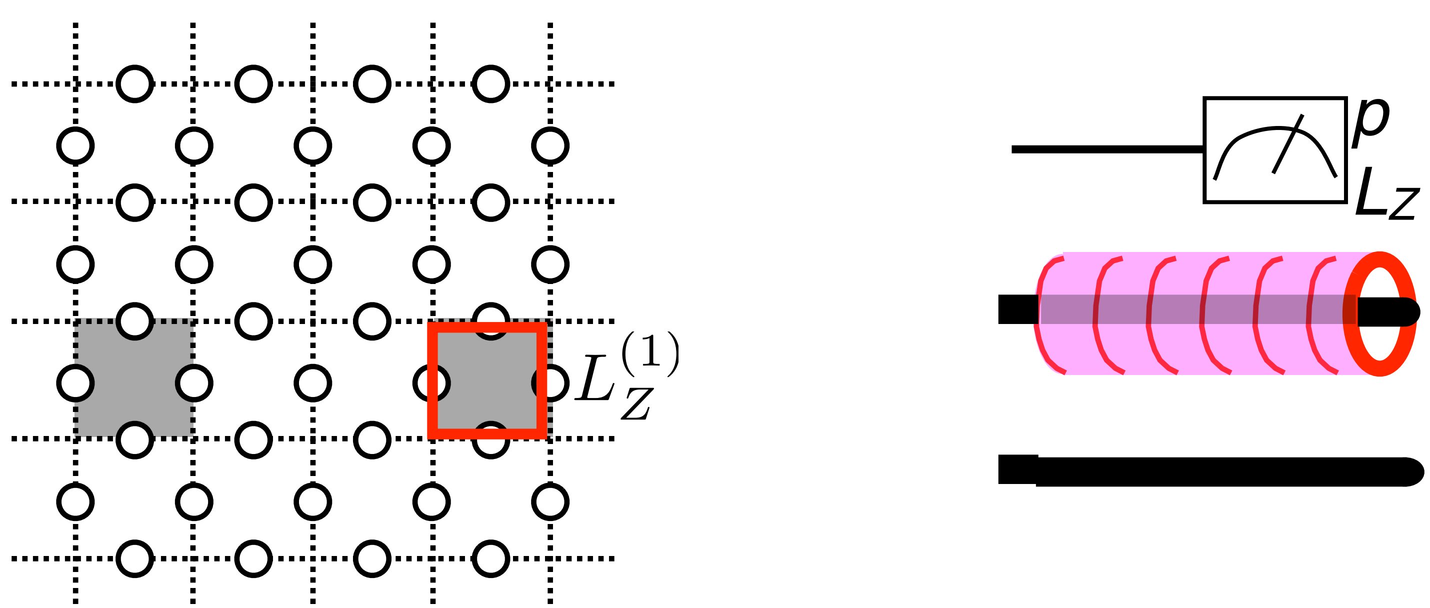
(2.18) ▶ Primal logical X measurement:

Since logical X operator is the tensor product of X on the chain which connects the primal defect pair, physical Pauli X measurements tell us logical X measurement outcome.



(2.19) ▶ Primal logical Z measurement:

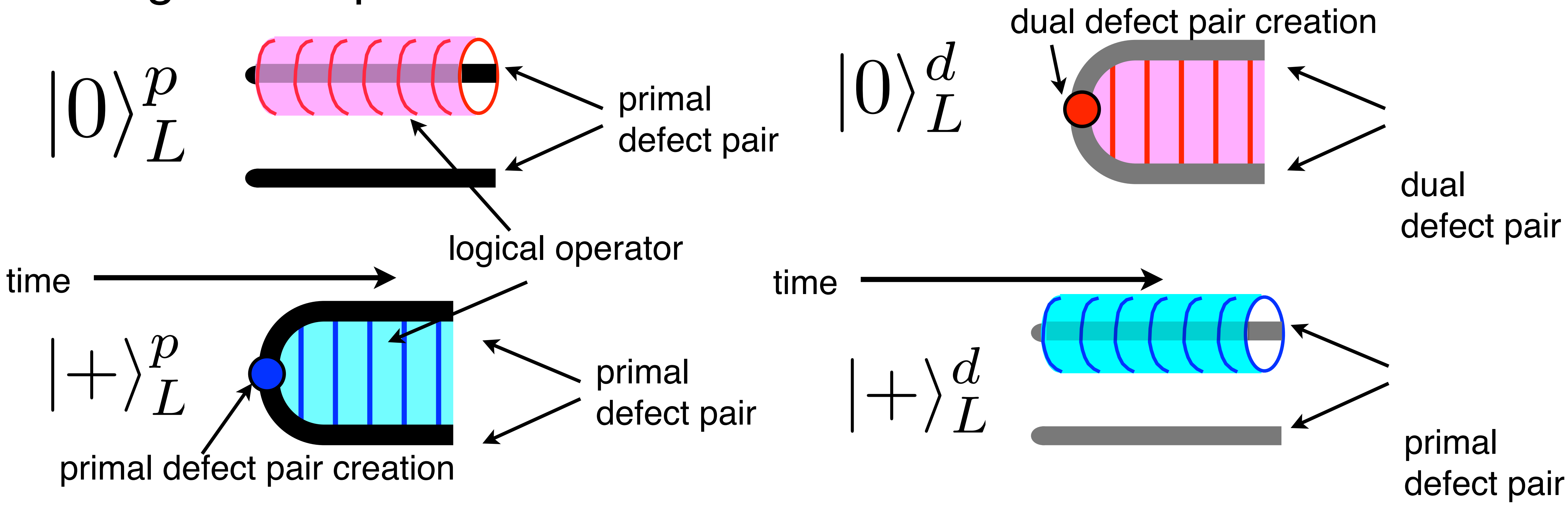
Stabilizer measurement of the removed face operator gives logical Z measurement outcome.



Diagram

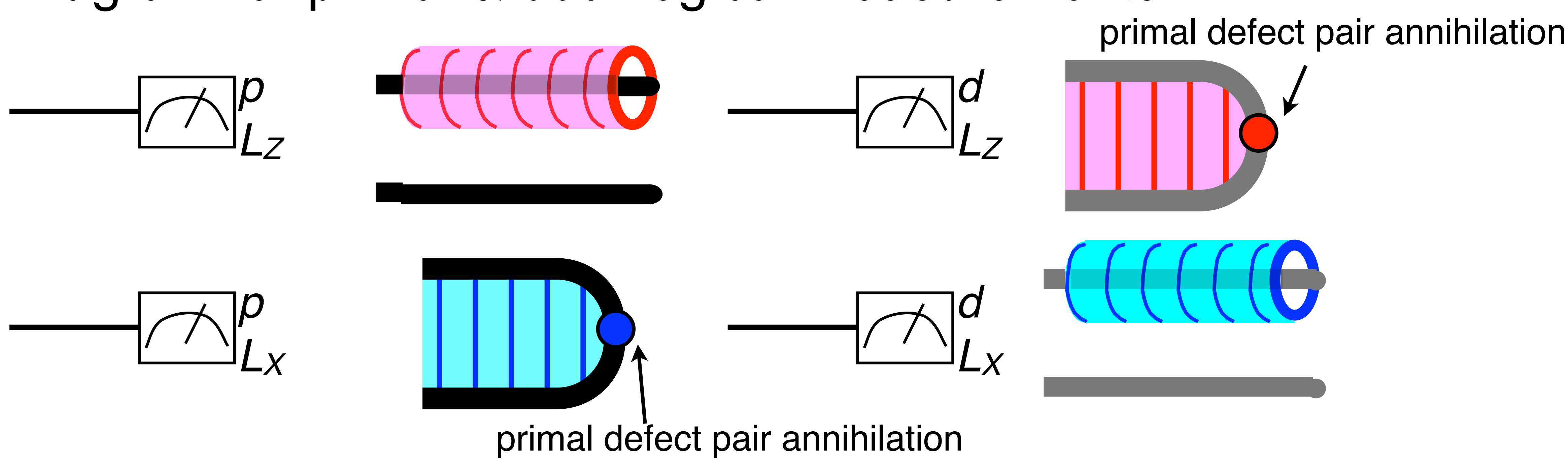
(2.20)

▶ Diagram for primal & dual defect creation:



(2.21)

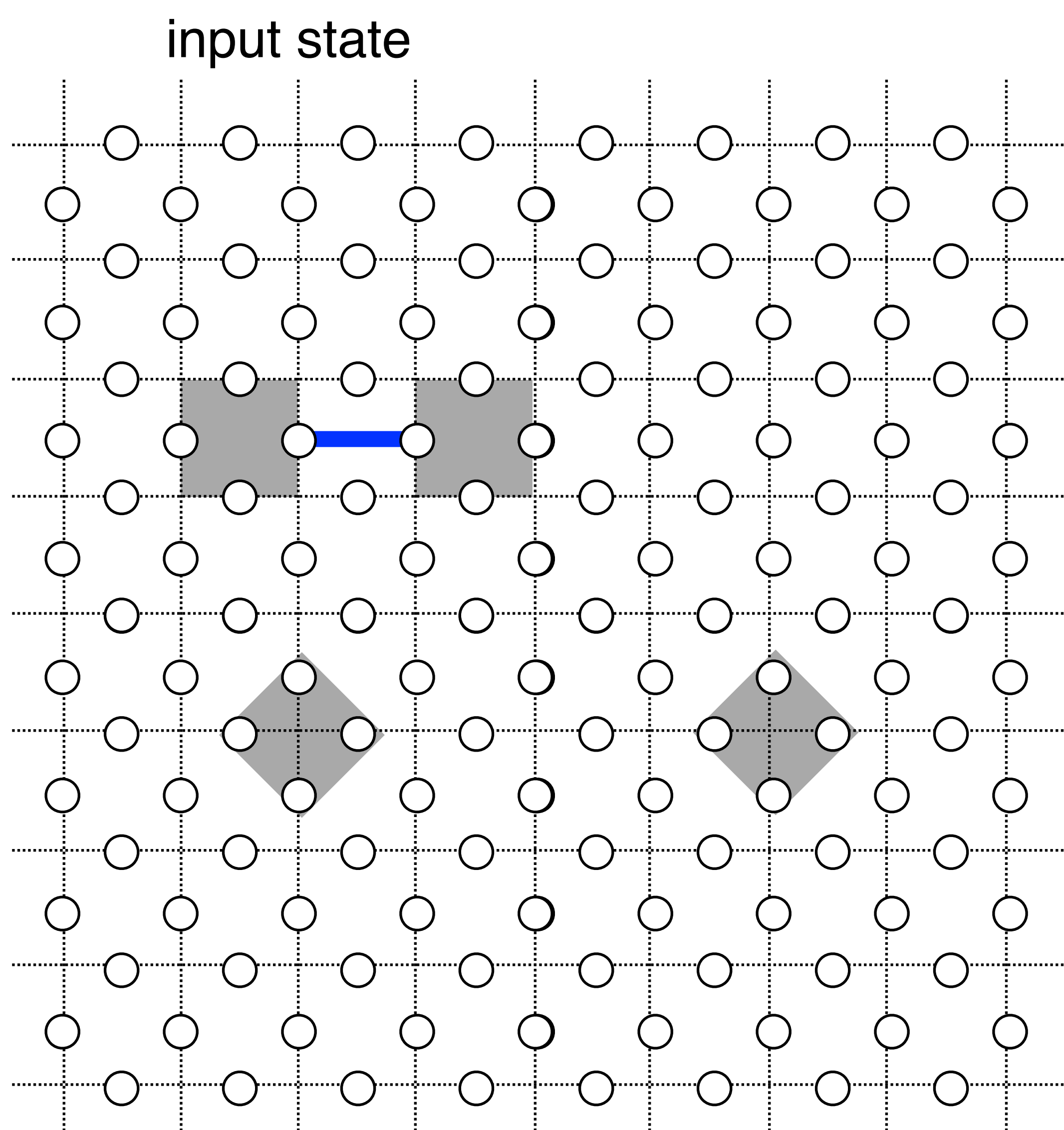
▶ Diagram for primal & dual logical measurements:



CNOT gate by braiding

(2.22) ▶Braiding the primal defect around the dual defect:

Let us first consider the time evolution of logical X operators under the braiding.



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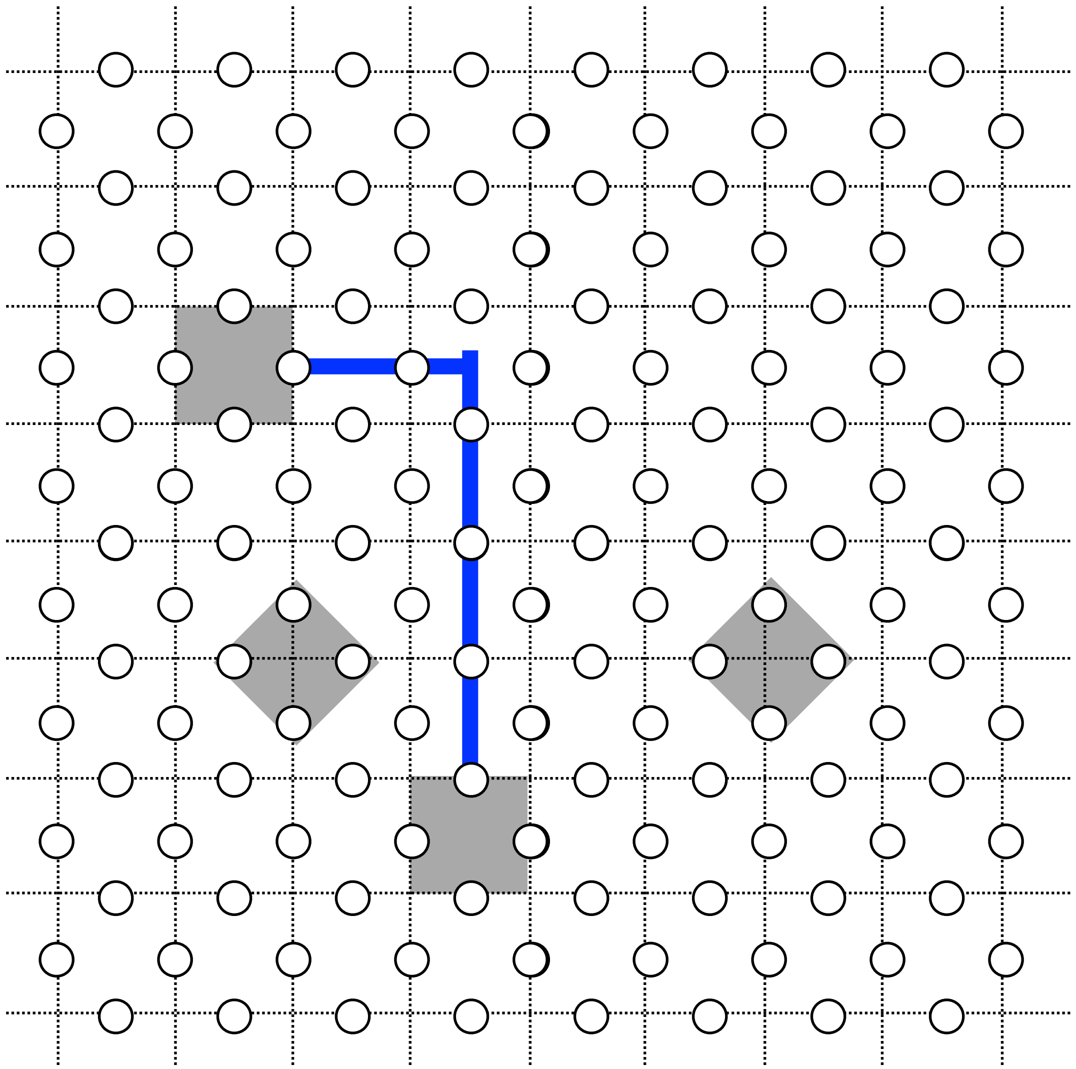

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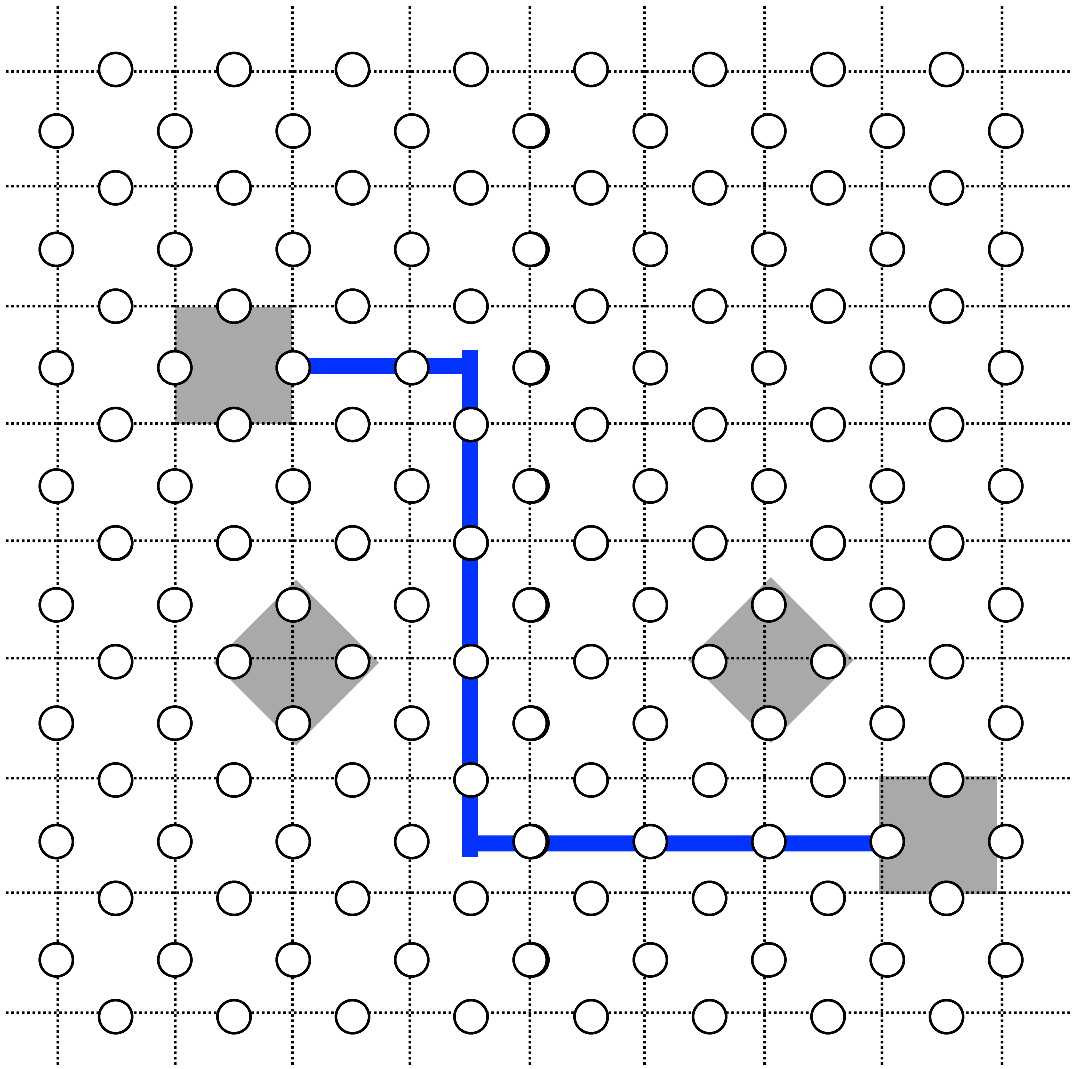

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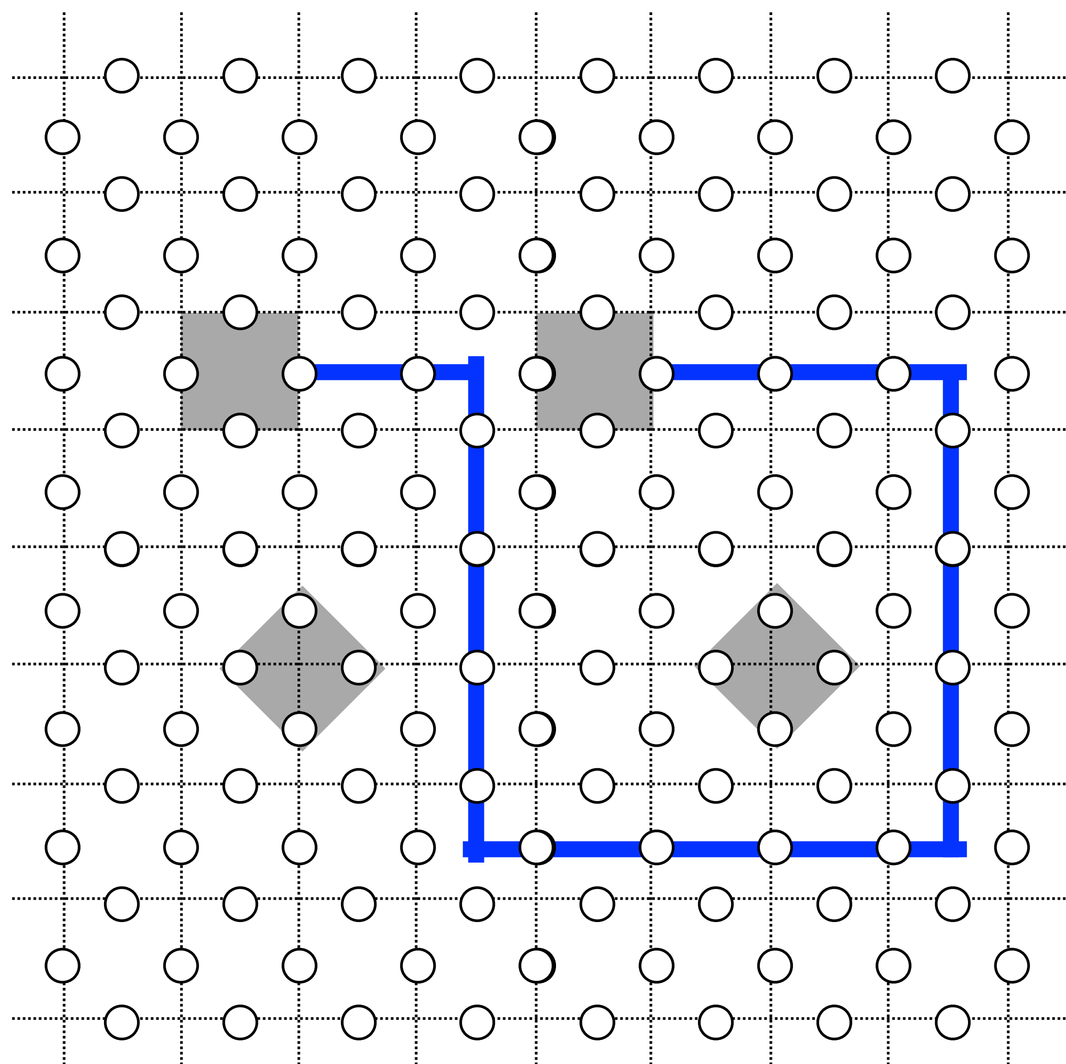

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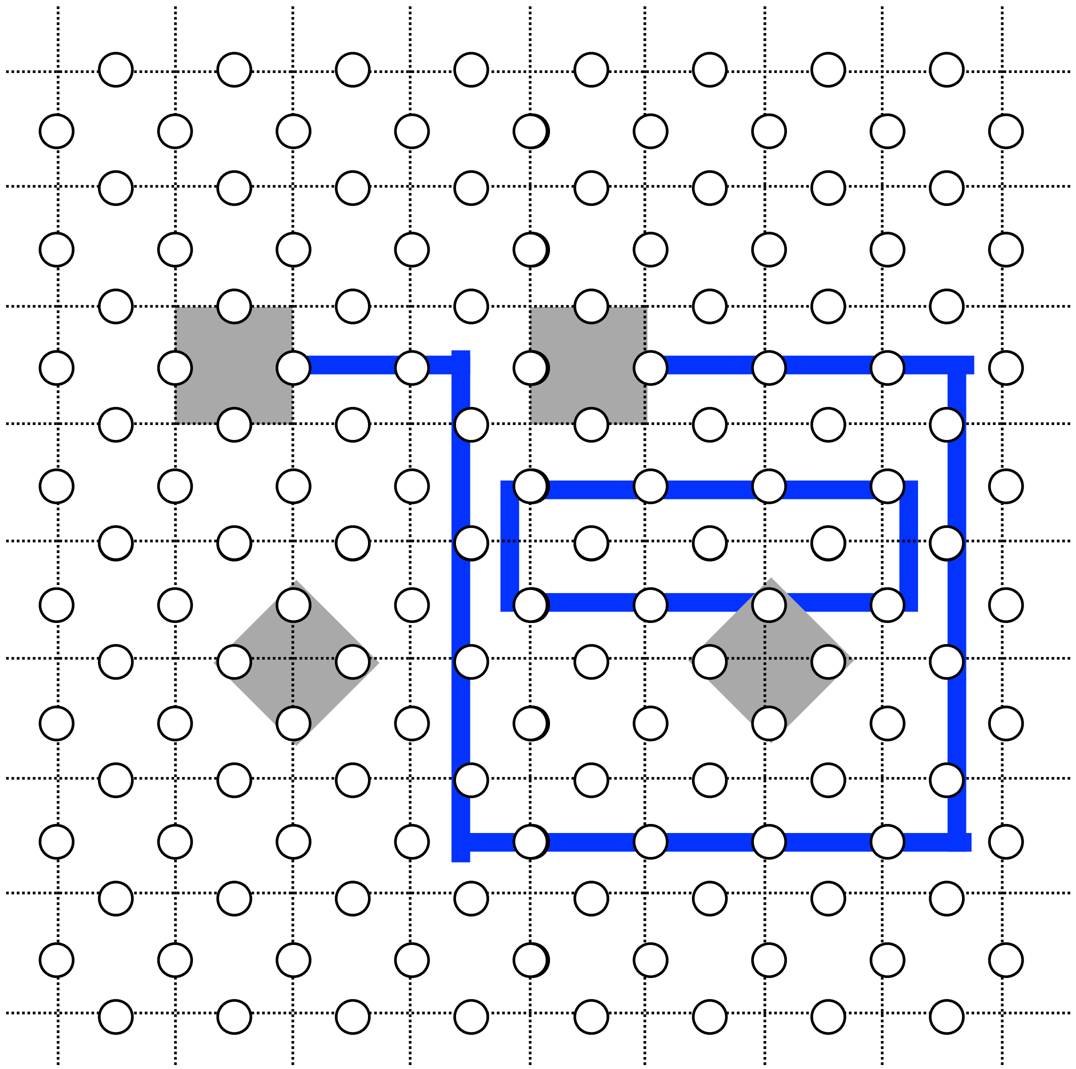

multiplication of loop operator
does not change the code state

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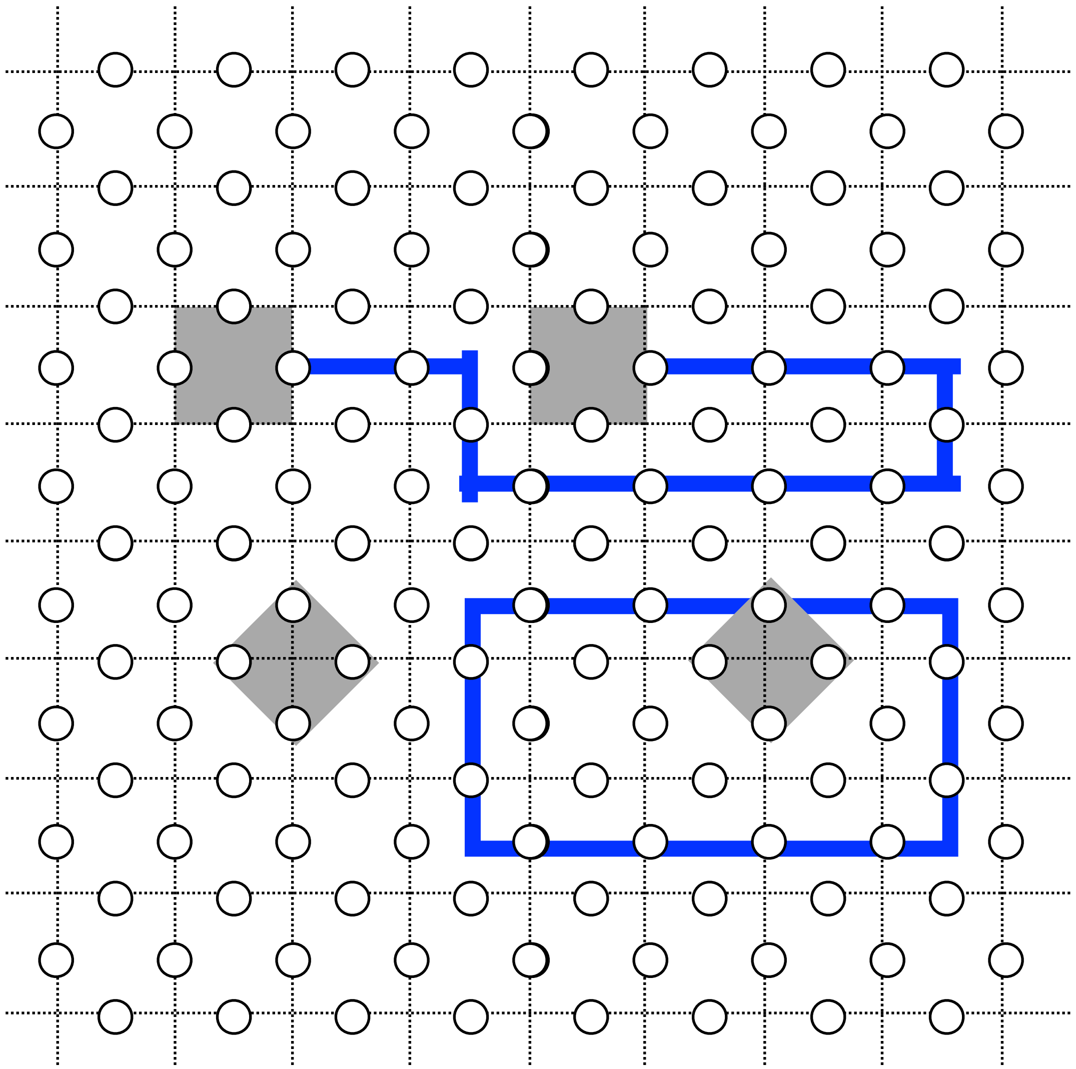

CNOT gate by braiding

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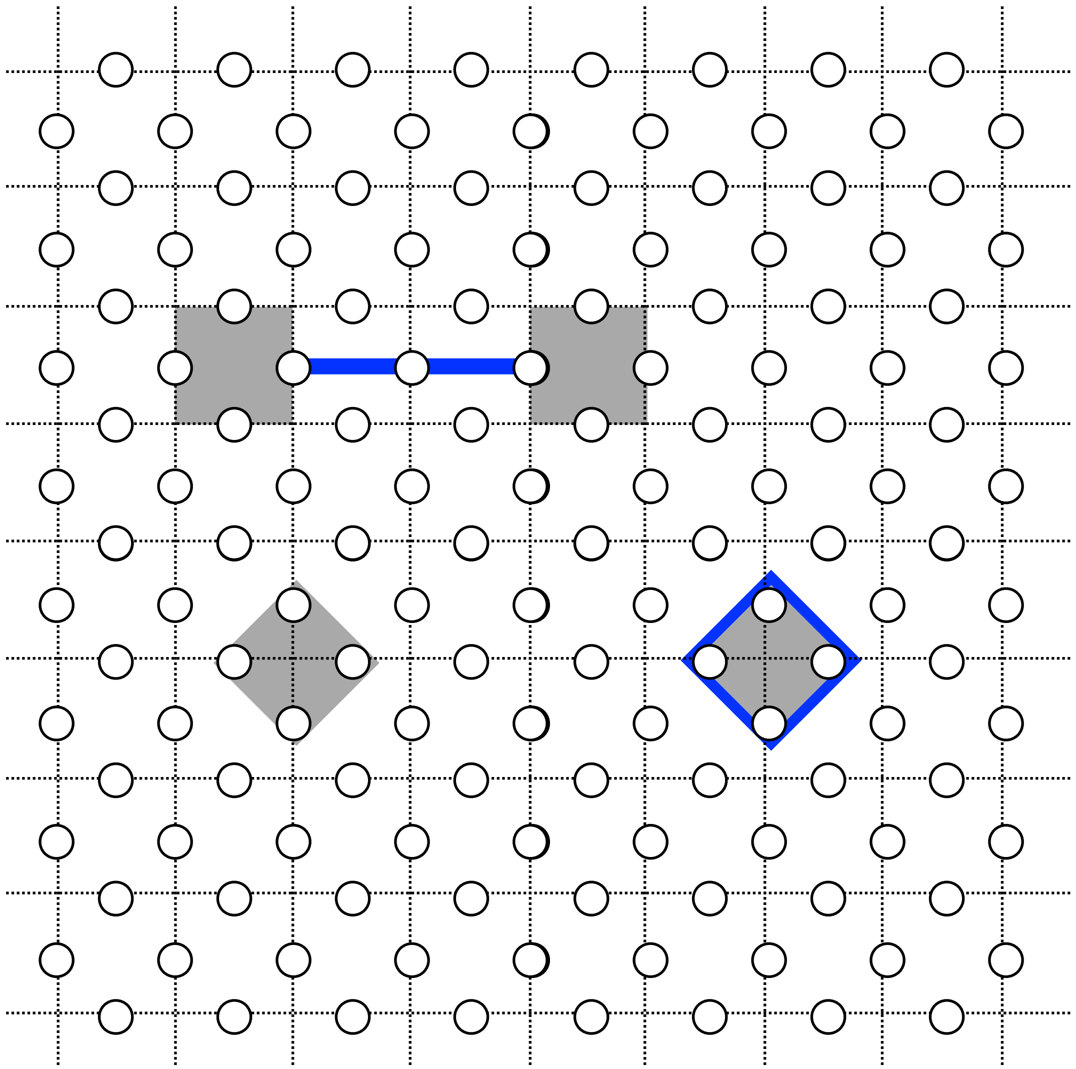


contraction does not change the logical operator

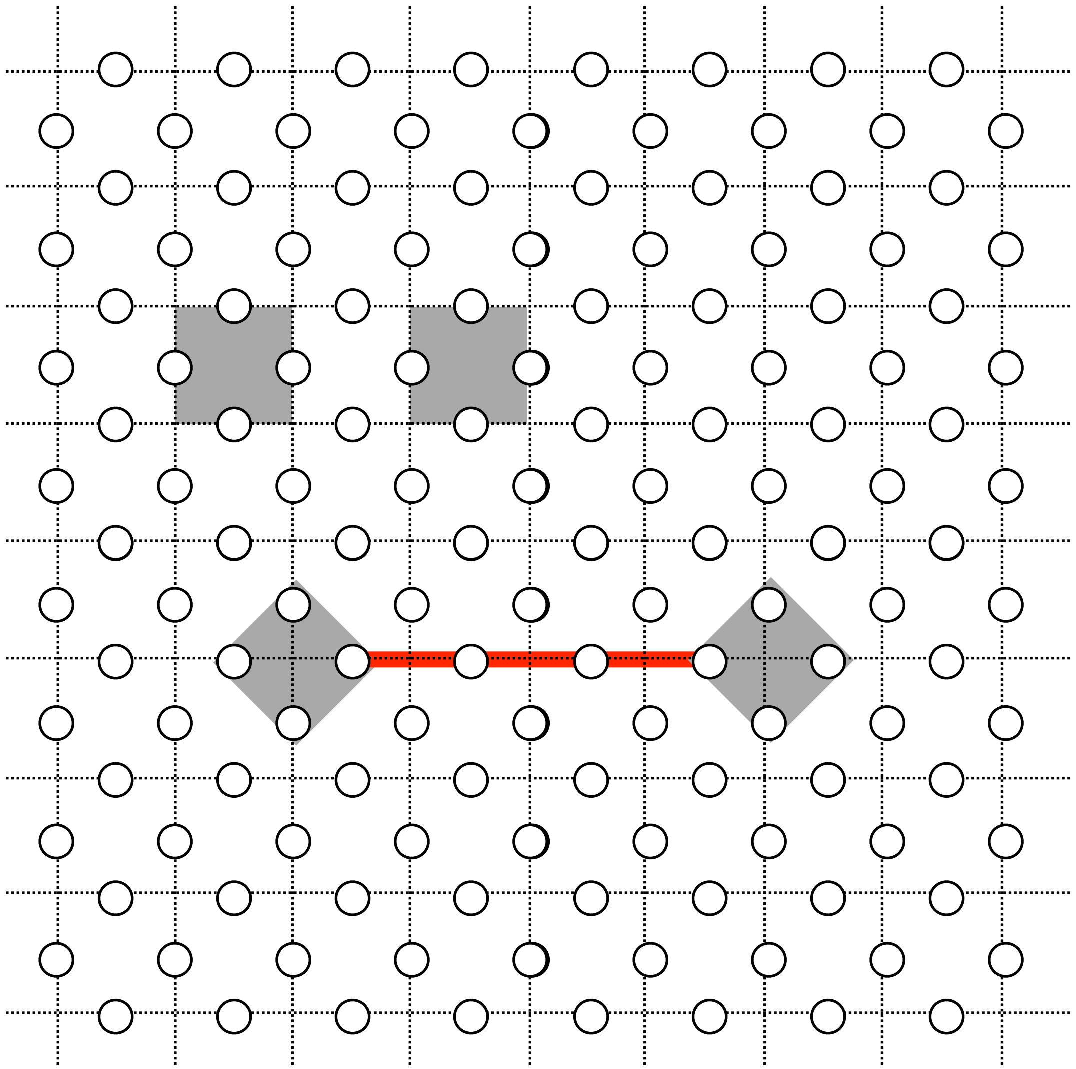
(2.23) $L_X^p \otimes I^d$ is transformed to $L_X^p \otimes L_X^d$ by the braiding.

CNOT gate by braiding

(2.24) ▶Braiding the primal defect around the dual defect:

Next we consider the time evolution of logical Z operators under the braiding.

input state



CNOT gate by braiding

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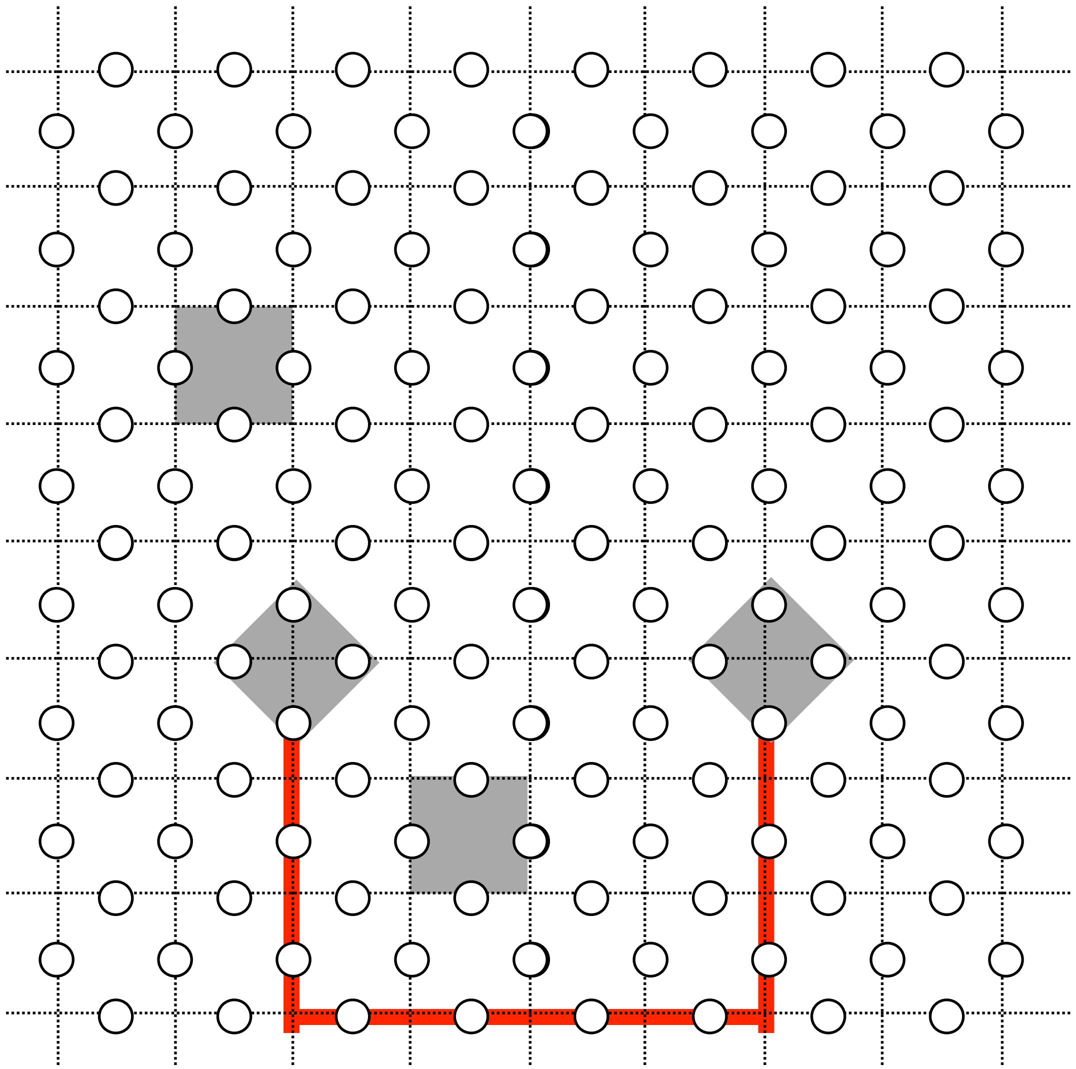

CNOT gate by braiding

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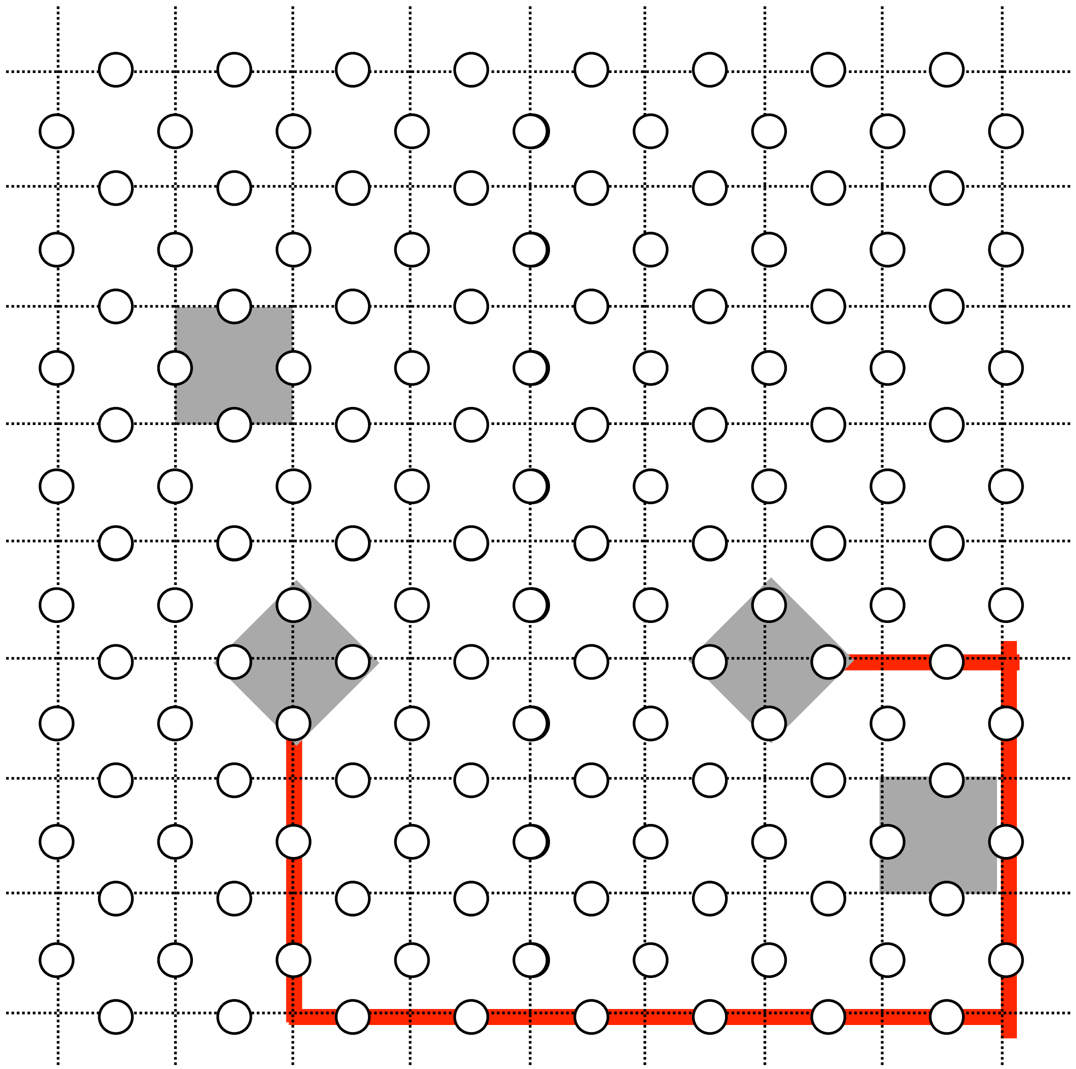

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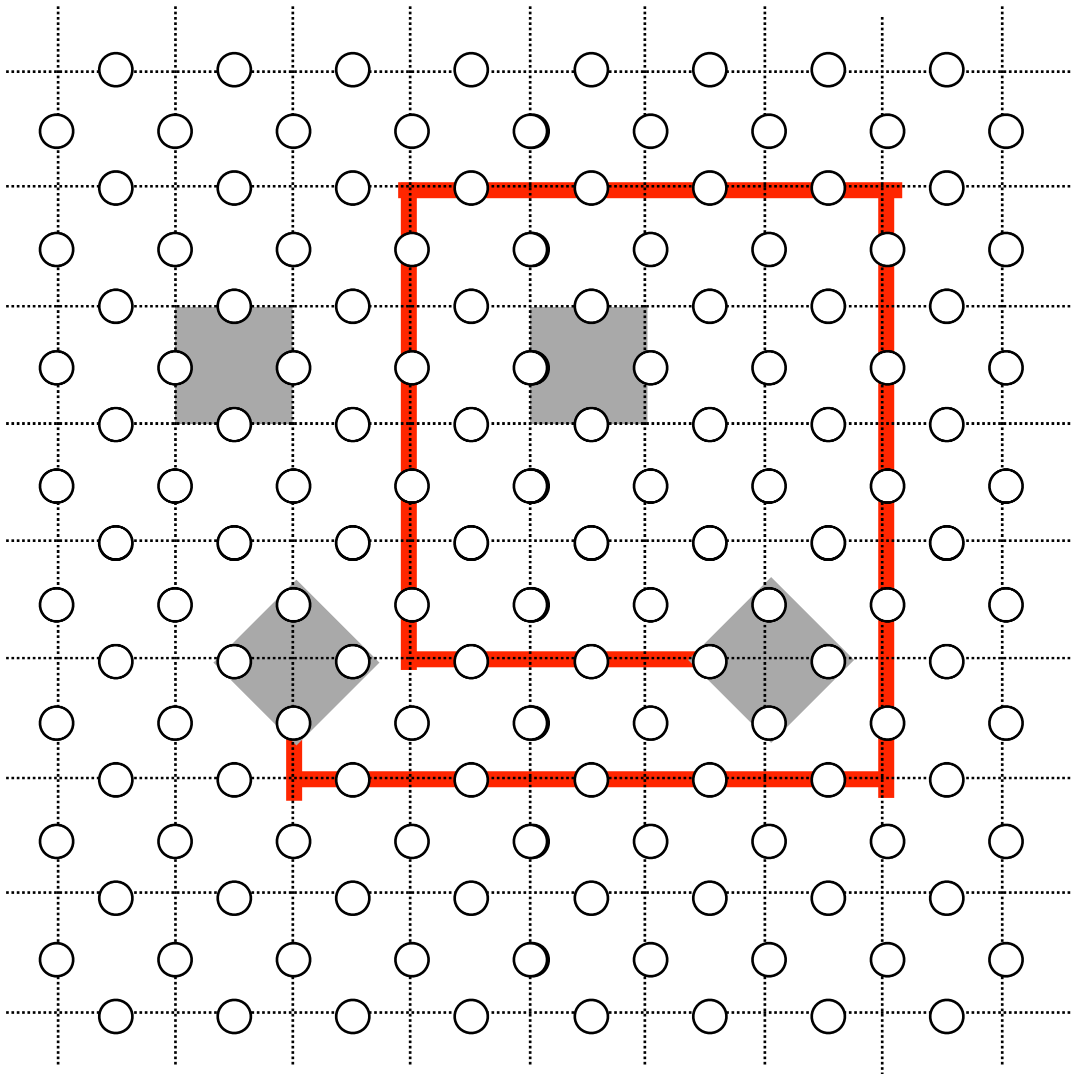

CNOT gate by braiding

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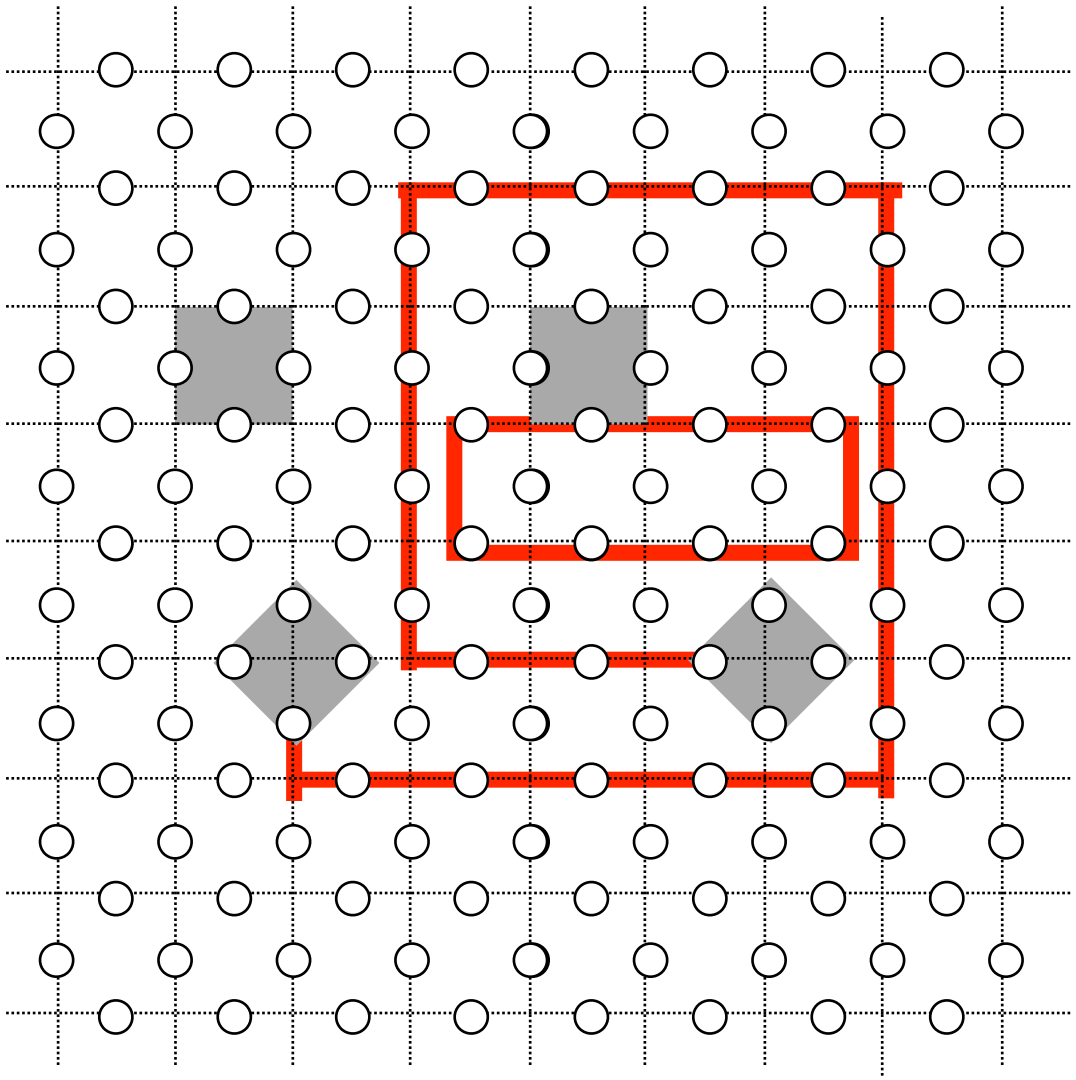

multiplication of loop operator
does not change the code state

CNOT gate by braiding

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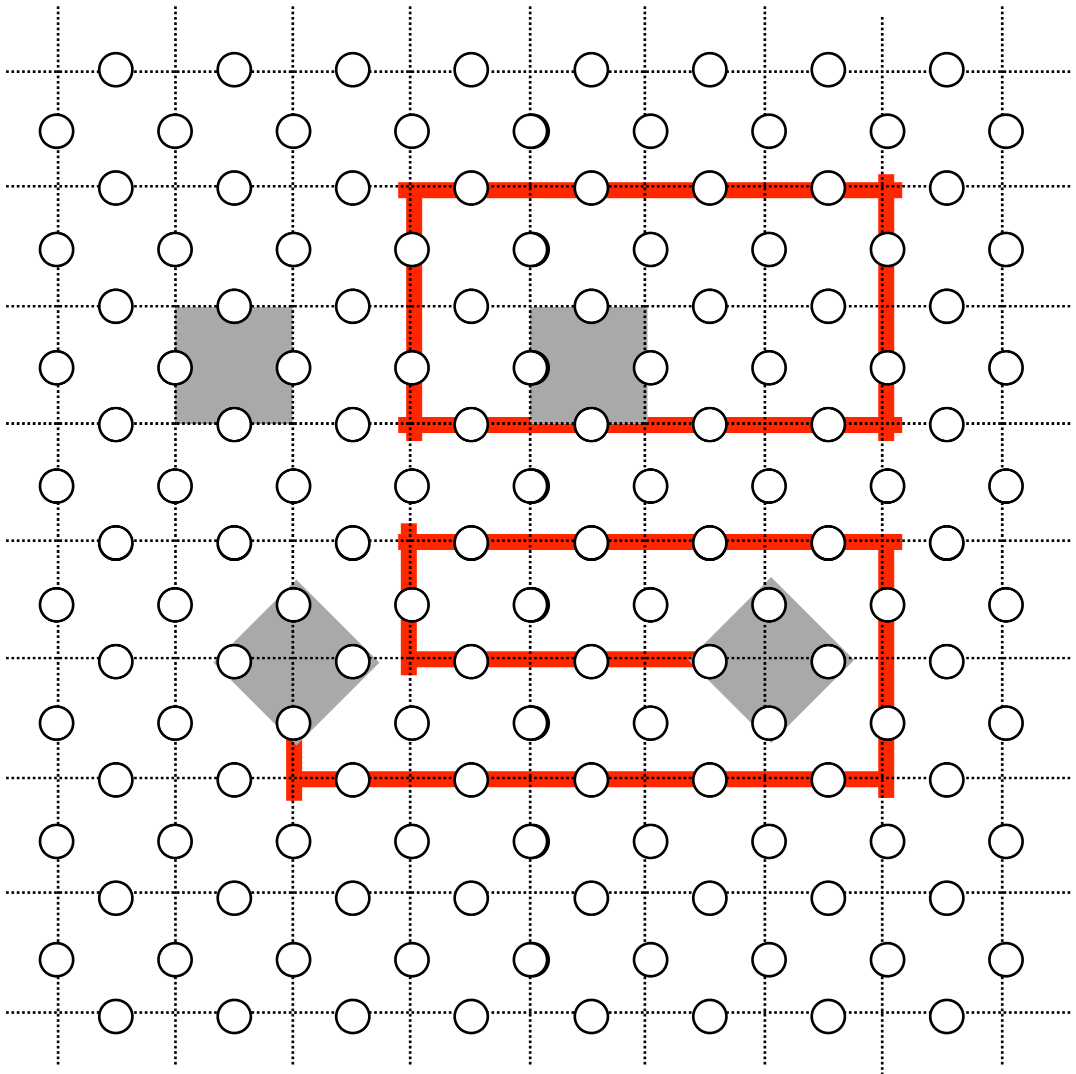
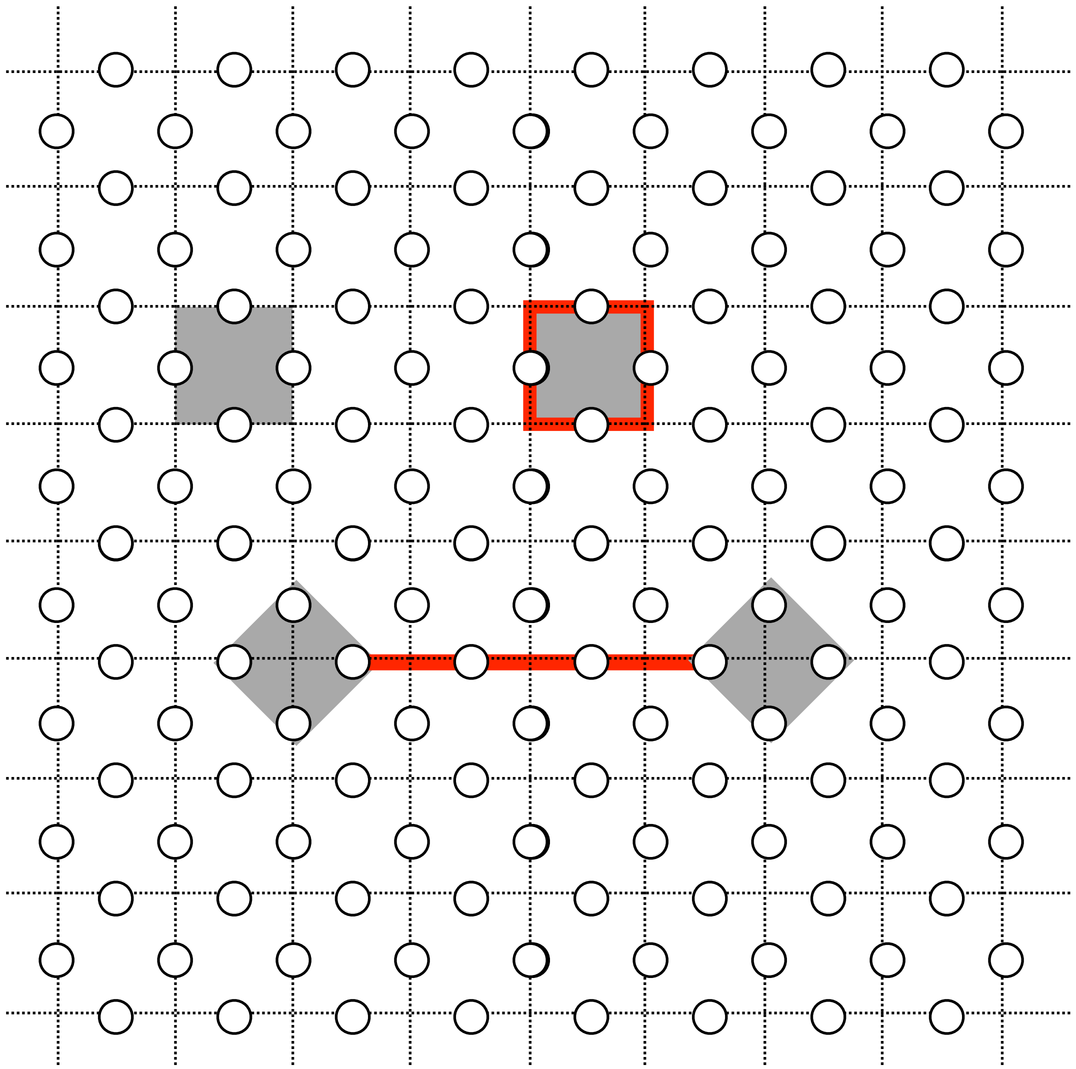


CNOT gate by braiding

(2.24) ▶Braiding the primal defect around the dual defect:

Next we consider the time evolution of logical Z operators under the braiding.

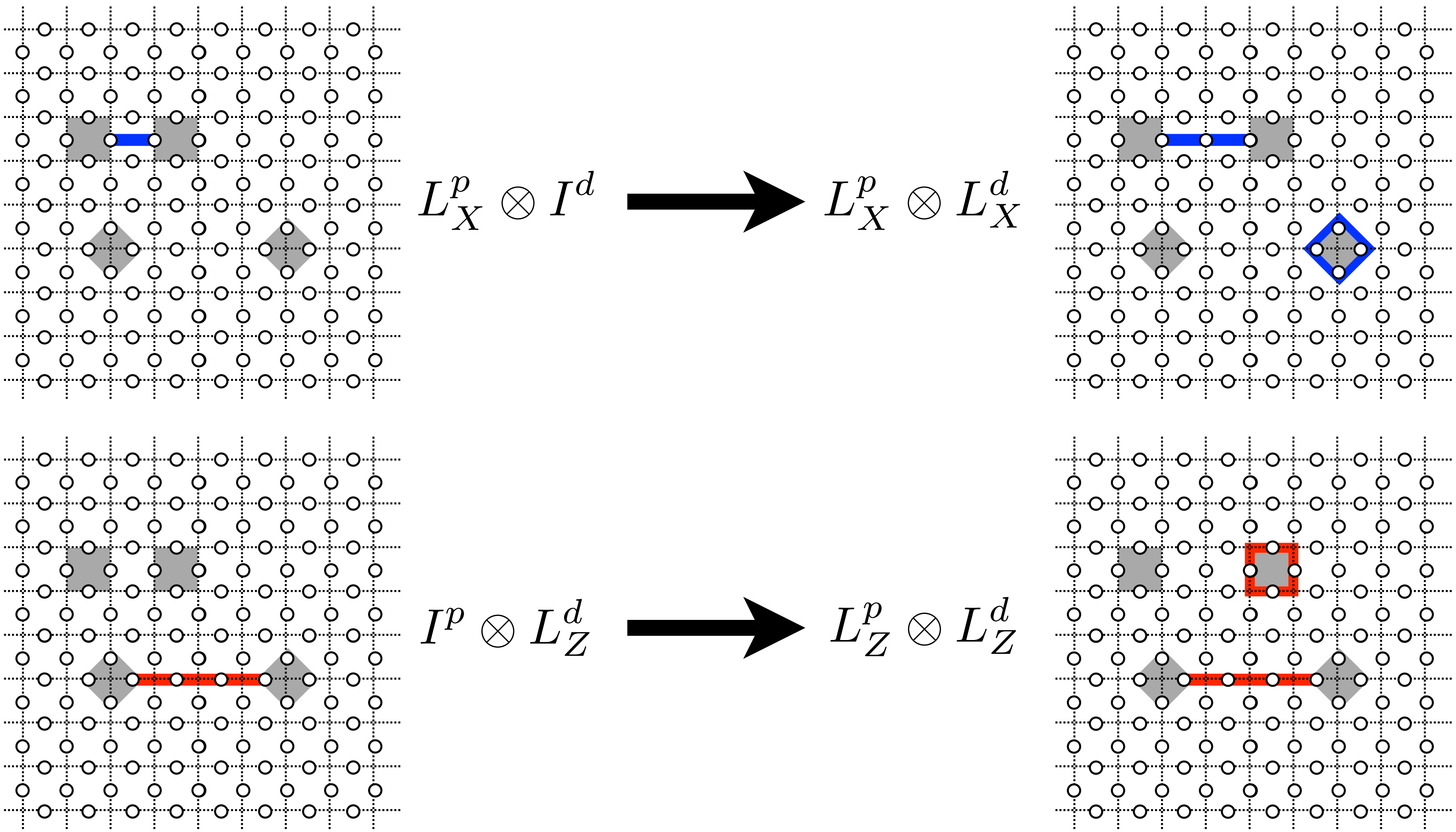
output state



contraction does not change the logical operator

(2.25) $I^p \otimes L_Z^d$ is transformed to $L_Z^p \otimes L_Z^d$ by the braiding.

CNOT gate by braiding

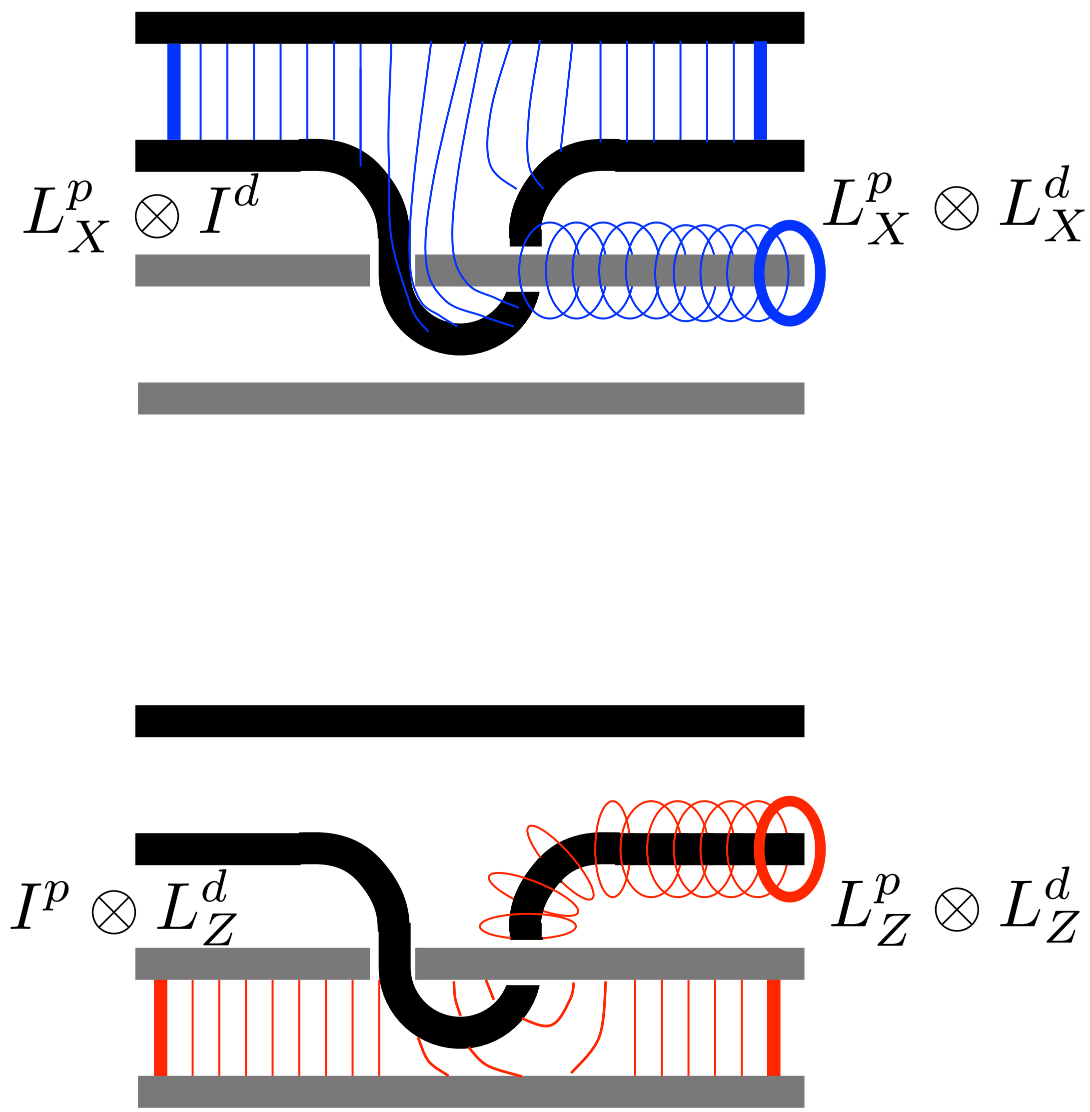


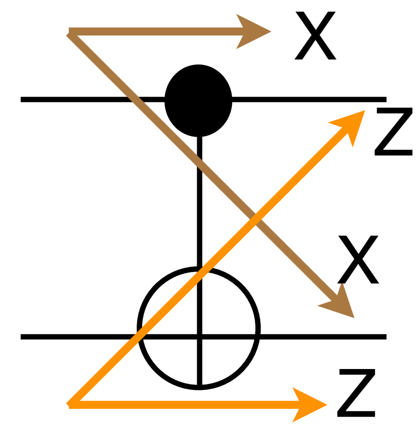
(2.26) ▶CNOT gate by braiding:

The transformation rule is equivalent to that of CNOT gate (1.7),which indicates that the braiding operation acts as the CNOT gate for primal (control) and dual (target) qubits.

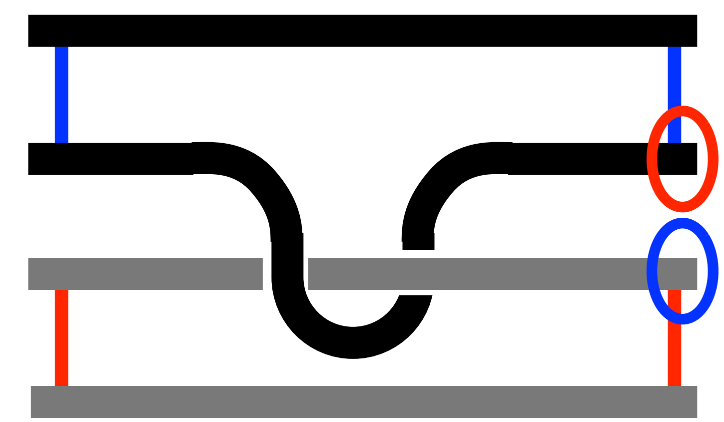
CNOT gate by braiding

(2.27) ▶ Diagram for the CNOT gate:





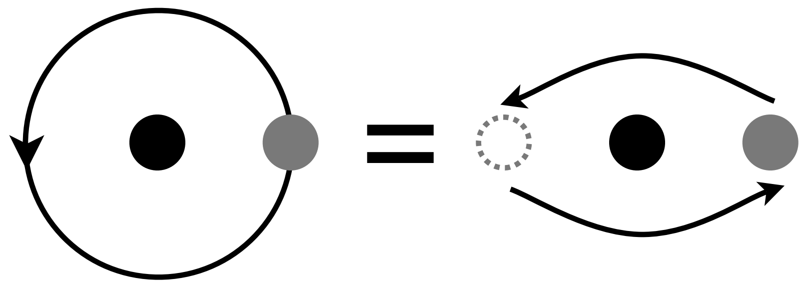
circuit diagram



topological diagram

Why “Abelian” anyon supports non-Abelian operations?

(2.28) ▶Braiding & anyon:



braiding = 2 × swap

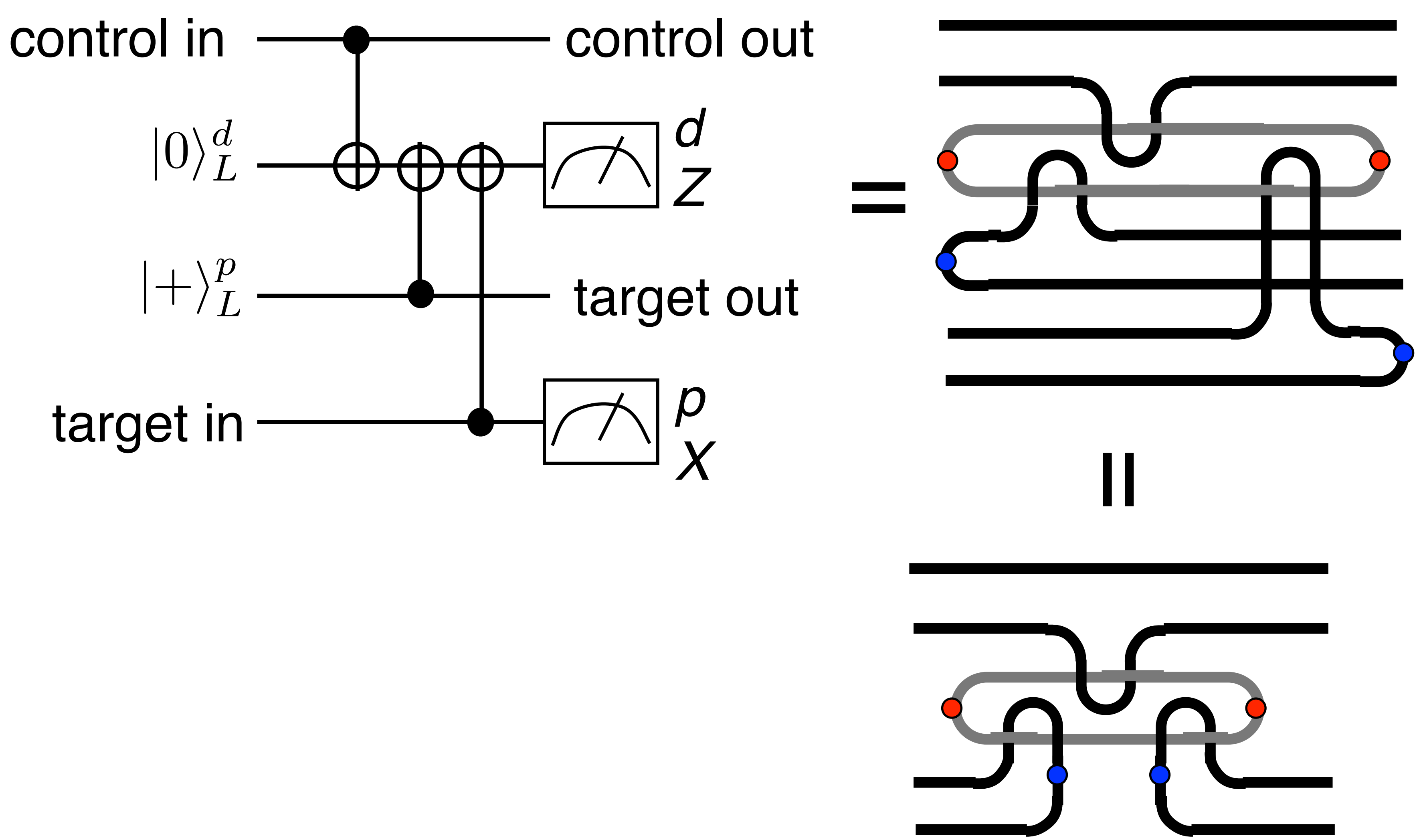
Thus braiding operations of boson or fermion result in trivial operations.

Since braiding of the defect change the code state, they are anyons.

The defect is Abelian anyon, since the control qubit of the CNOT is always the primal qubit.

Why “Abelian” anyon supports non-Abelian operations?

(2.29) ▶How to support non-Abelian operations by using Abelian anyons:



This can be understood that Abelian anyons support non-Abelian operations by changing the topology of surface!

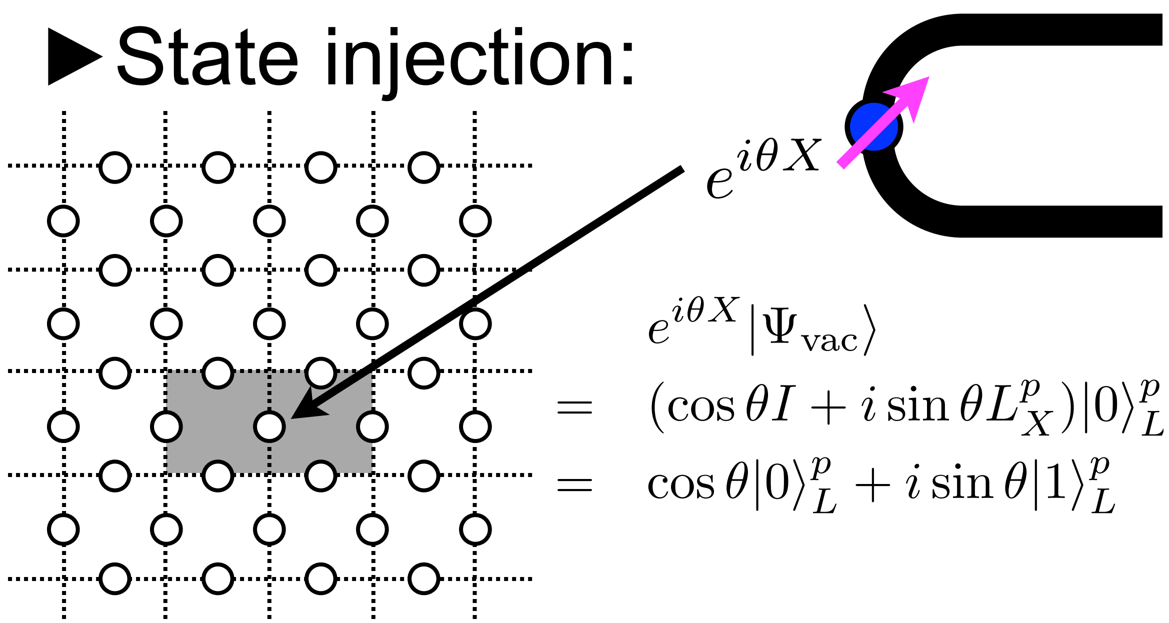
Universal topological quantum computation

(2.30) ## ▶Non-Clifford gates:

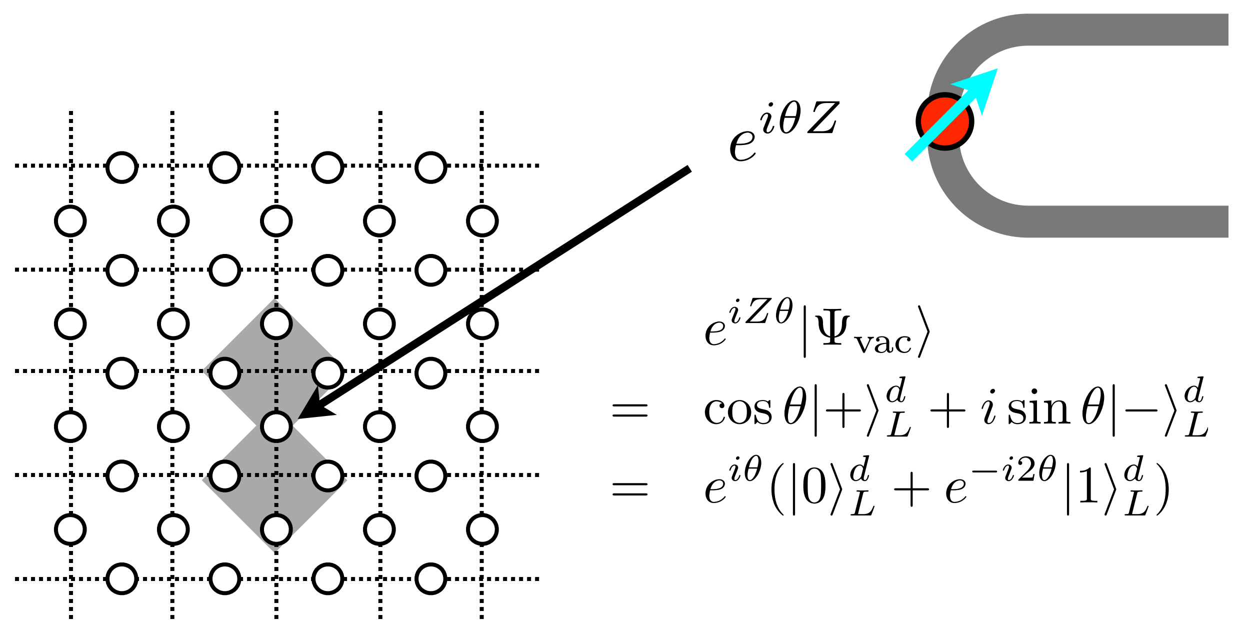
So far, we have Pauli basis preparation, CNOT gate, Pauli basis measurements, which allow us an arbitrary Clifford operations.

However, quantum computation with Clifford operations can be efficiently simulated by classical computer. Thus non-Clifford gates are necessary.

(2.31) ## ▶State injection:



$$e^{i\theta X}|\Psi_{\rm vac}\rangle = (\cos\theta I + i\sin\theta L_X^p)|0\rangle_L^p = \cos\theta|0\rangle_L^p + i\sin\theta|1\rangle_L^p$$

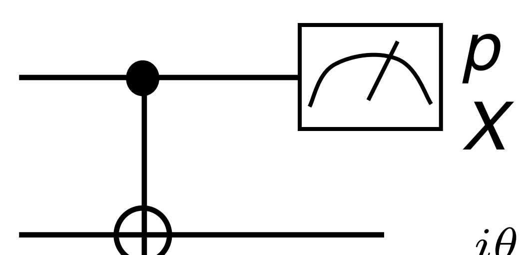


$$e^{iZ\theta}|\Psi_{\rm vac}\rangle = \cos\theta|+\rangle_L^d + i\sin\theta|-\rangle_L^d = e^{i\theta}(|0\rangle_L^d + e^{-i2\theta}|1\rangle_L^d)$$

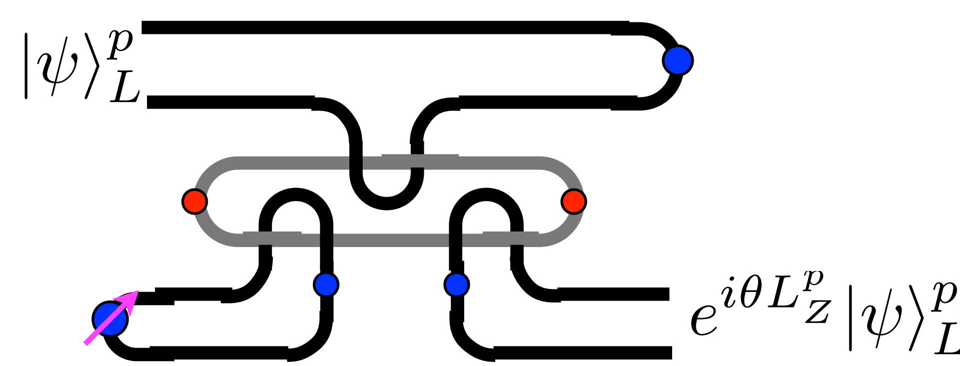
(2.32) ## ▶One-bit teleportation for non-Clifford gate

$|\psi\rangle_L^p$

$\cos\theta|0\rangle_L^p + i\sin\theta|1\rangle_L^p$

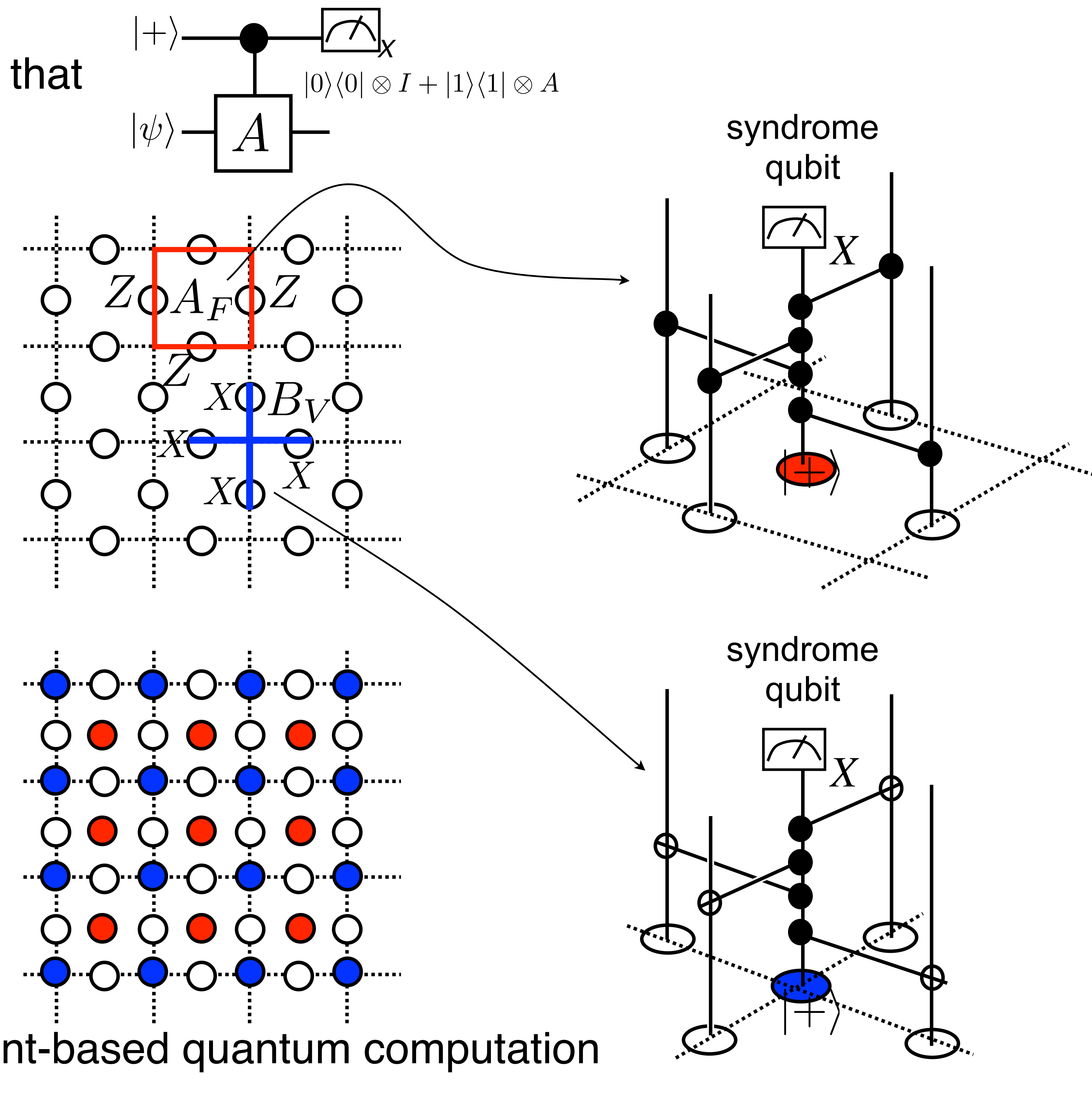


$e^{i\theta L_Z^p}|\psi\rangle_L^p$



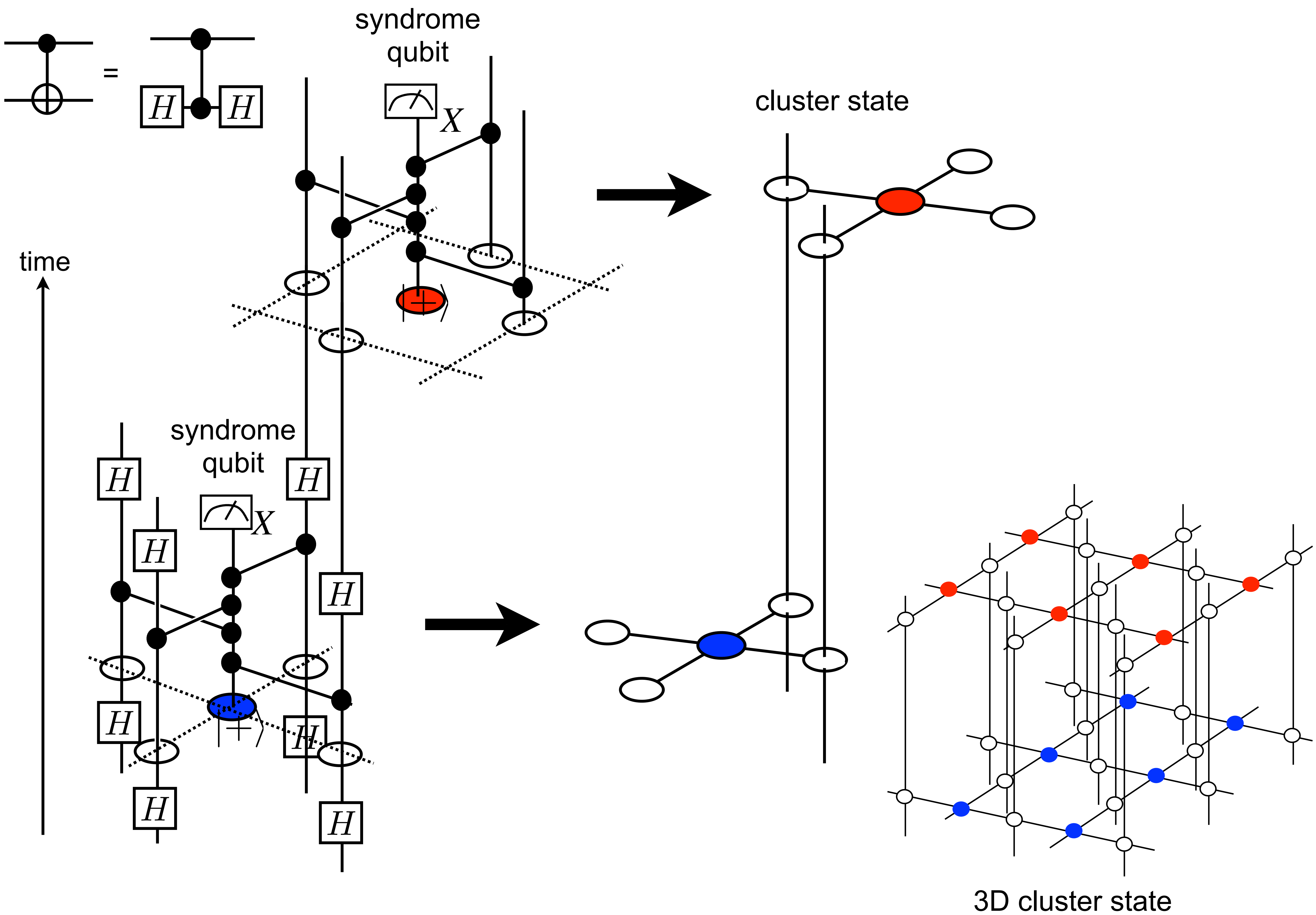
Fault-tolerant QC in 2D

Recall that



measurement-based quantum computation

Fault-tolerant QC in 3D



応用例

Fault-tolerant quantum computation with high noise threshold ~5%:

KF & K. Yamamoto, Phys. Rev. A **82**, 060301(R) (2010)
[関連研究：KF & K. Yamamoto, Phys. Rev. A **81**, 042324 (2010)]

Fault-tolerant quantum computation with probabilistic two-qubit gate:

KF & Y. Tokunaga, Phys. Rev. Lett. **105** 250503 (2010)

Topological quantum computation on the thermal state of spin-3/2 system:

KF & T. Morimae, arXiv:1111.0919, to be appeared in PRA Rapid Com.
[関連研究：KF & T. Morimae, arXiv:1106.3377]

Topological blind quantum computation:

T. Morimae & KF, arXiv:1110.5460

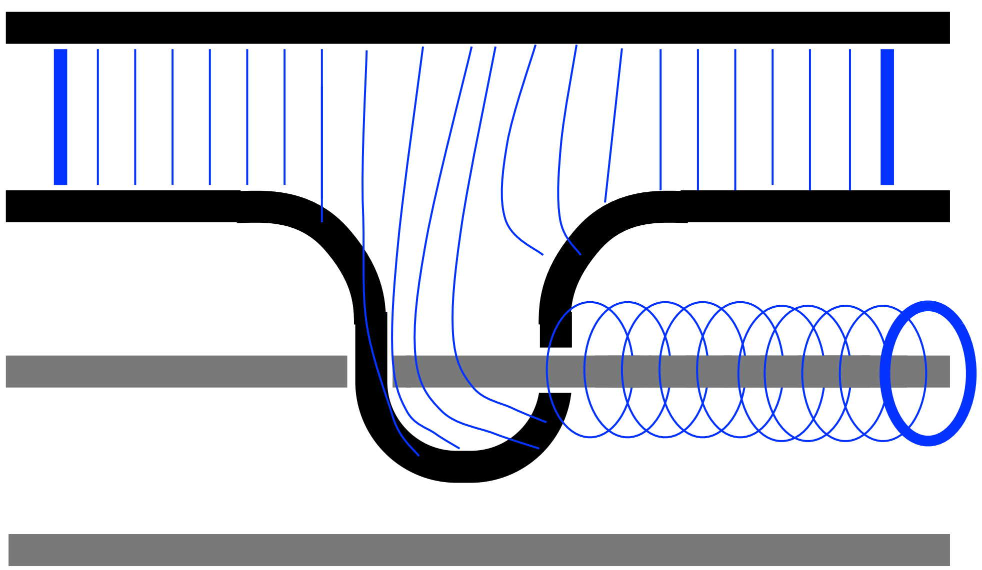
あらすじ

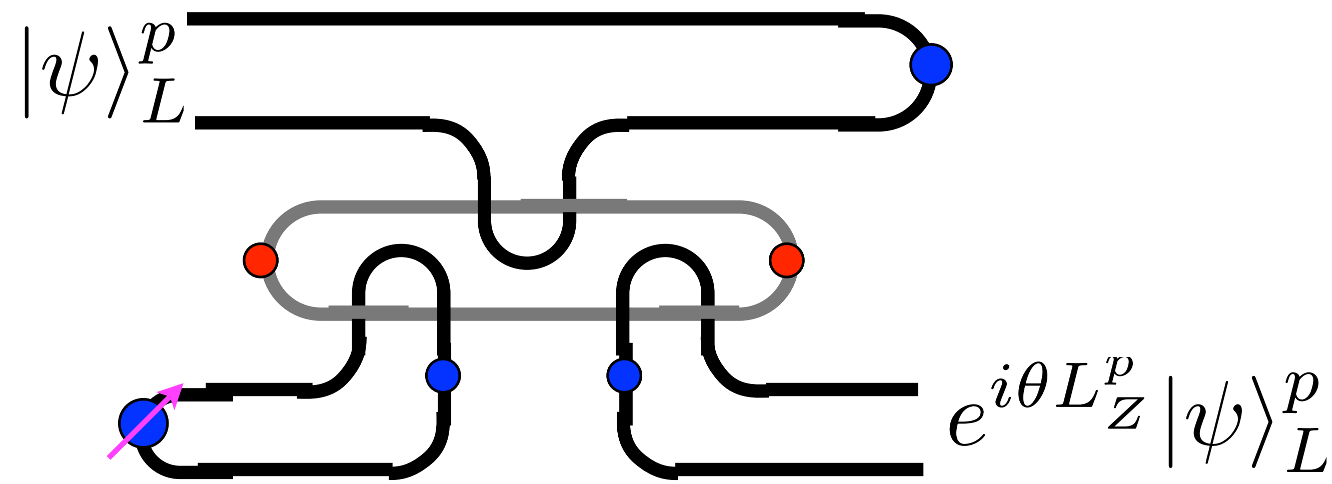
- ✓ セットアップ
  qubit, Pauli operator, Clifford gates
- ✓ スタビライザー形式
  stabilizer group, state, subspace, logical operator, code
- ✓ 量子誤り訂正
  syndrome measurement, indirect measurement
- ✓ トポロジカル量子メモリ
  surface code, trivial & non-trivial loop operator
- ✓ トポロジカル量子計算
  braiding, diagram, 2D, 3D

あらすじ

✓セットアップ

qubit, Pauli operator, Clifford gates





✓トポロジカル量子メモリ

surface code, trivial & non-trivial loop operator

✓トポロジカル量子計算

braiding, diagram, 2D, 3D

Gottesman-Knill's theorem

Theorem

If the input state is product states of eigenstates of Pauli operators, all unitary operations are Clifford gates, and the output state is measured in Pauli basis, then such quantum computation can be efficiently simulated by classical computer.

Input state is a stabilizer state. Each stabilizer generator of the *n*-qubit system can be represented by 2*n*-bit.

(1.14) eg) $XX \rightarrow (1,1|0,0) \quad ZX \rightarrow (0,1|1,0)$

i th X is i th 1, j th Z is $(n+j)$th 1.

Since Clifford gates map a stabilizer state to another stabilizer state, they can be expressed as a linear map of $\mathbf{Z}_2^{2n}$. Thus stabilizer generators of the output state can be easily calculated by using classical computer.

Furthermore, the probability distribution of the output state under the Pauli basis $A_i = I, X, Y, Z$ measurement is easily calculated by using classical computer.

$$\begin{aligned} p(\nu_1, \cdots, \nu_i) &= \langle \Psi | \prod_{i=1}^{n} \frac{I + (-1)^{\nu_i} A_i}{2} | \Psi \rangle \\ &= \langle \Psi | \sum_{(\zeta_1, \cdots, \zeta_n)} \frac{1}{2^n} \prod_{i=1}^{n} [(-1)^{\nu_i} A_i]^{\zeta_i} | \Psi \rangle \\ &= \langle \Psi | \sum_{*} \frac{1}{2^n} \prod_{i=1}^{n} [(-1)^{\nu_i} A_i]^{\zeta_i} | \Psi \rangle \end{aligned}$$

($\nu_i = 0, 1$ measurement outcome)

($\zeta_i = 0, 1$)

where $* := \left\{ (\zeta_1, \cdots, \zeta_n) | \prod_{i=1}^{n} \pm A_i^{\zeta_i}, \pm i A_i^{\zeta_i} \in \mathcal{S} \right\}$ ■